Language in mathematics teaching and learning: A research review

By Mary J. Schleppegrell, University of Michigan[i]

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Overview

The words "language and mathematics" can be thought of in two different ways: as referring to their relationship as systems of meaning-making and as referring to the role of language in the pedagogical context of mathematics classrooms. Both of these aspects of the relationship of language to mathematics are addressed in this review. Language has been a topic of discussion by mathematics educators for more than a generation. Stimulated by seminal work in the 1970s and 1980s (e.g., Halliday, 1978; Pimm, 1987), the notions that mathematics is itself 'language-like' and that language issues are an important focus in classroom teaching have been consistent themes in mathematics education research. The idea that communication and discourse are integral to learning mathematics is now explicit in mathematics standards, and researchers have focused on the role of language and interaction in the mathematics classroom from a variety of perspectives that go by labels such as *constructivist*, *sociocultural*, and more recently, *semiotic*. This review discusses trends in research on language and mathematics, focusing in particular on recent work that recognizes the role of language in constructing knowledge and that has described the features of the mathematics register in comprehensive ways, recognizing its multisemiotic nature (e.g., Lemke, 2003; O'Halloran, 2005).

Although in the past it has been suggested that mathematics draws less on language than other school subjects do, that view has now been discredited. In fact, it has become clear that language plays as important a role in mathematics learning as in learning other school subjects. Mathematics educators have gone beyond vocabulary and word-based thinking about language, as words alone do not suffice in identifying and describing the language challenges of mathematics (although in practice even some current research still focuses on language as meaning at the level of the word or phrase). The notions of mathematics as *discourse* and students as being apprenticed into particular ways of *doing mathematics* in particular discursive contexts are now gaining prominence in mathematics education research.

Every school subject is constructed in language, but the forms and patterns language takes vary from discipline to discipline. The language through which mathematics is constructed tends to be conceptually dense, interpersonally alienating, and highly structured textually in unfamiliar ways. A key challenge is that mathematics incorporates a symbolic language that developed out of natural language and also uses visual display to construct complex meanings. Students need to be able to work simultaneously with all aspects of this multi-semiotic system–natural language, the language of mathematics symbolism, and the visual semiotic constructed in the graphs, charts, and diagrams that are integral to mathematical reasoning. Recent research on language and mathematics using semiotic approaches and linguistic tools is providing insights into the ways these meaning-making systems interact in construing mathematical knowledge and is exploring the ways students are positioned by language in mathematics classrooms. This research offers possibilities for new understanding of the role of the teacher in supporting students' development of mathematical knowledge.

The first section of this review discusses different perspectives on the conceptual relationship between language and mathematics and describes semiotic approaches that recognize the importance of language in constructing mathematical knowledge. The second section focuses language in the mathematics classroom, where researchers have described the dilemmas teachers face in moving between everyday and technical language in negotiating mathematics meaning in classrooms with diverse students. The third section describes studies that use functional linguistics perspectives to analyze language in mathematics classrooms using rich theoretical frameworks and drawing on rigorous analytic tools. The fourth section addresses the potential for teacher education to promote greater understanding of the role of language in mathematics education, and the final section provides a summary of recommendations for future research that emerge from this review.

Language and mathematics conceptualized

The relationship between language and mathematics was developed in a frequently-cited seminal work by Pimm (1987), who used the metaphor that *mathematics is a language* and summarized a set of linguistic phenomena that characterize the language of mathematics. These included mathematical usage of ordinary words with different meanings (*difference, odd/even*), technical terms particular to mathematics, and different ways of interpreting words and phrases in mathematics. But Pimm also pointed out that mathematics is like language in that it is a system for making meaning. He drew educational implications from this understanding, suggesting that mathematics instruction needs to emphasize meaning and not form so that it is not treated as a kind of blind symbol manipulation that cuts students off from the real meaning of mathematics. Stimulated by the work of Pimm and others, the mathematics education community took up the notions of *discourse* and *communication* as central to mathematics learning. At the same time, individualist orientations that were proving inadequate for understanding mathematics teaching and learning stimulated interest in the notion of mathematics as social practice (Solomon, 1989).

Two perspectives on the relationship between language and mathematics can be characterized as *constructivist* and *socioculturalist.* While researchers define the approaches and position themselves relative to them in different ways, the constructivist view comes out of a

Piagetian perspective, that everyone constructs internal representations, or mental structures, for him/herself. Cobb & Yackel (1998) characterize the constructivist perspective as interpretive, seeing knowledge as actively constructed by the student in interaction with the environment. Constructivists interpret the way students talk about mathematics to investigate students' development of mathematical knowledge (see also Laborde, Conroy, De Corte, Lee, & Pimm, 1990). Sociocultural perspectives (e.g., Adler, 1997) focus on discursive practices and the social engagement of students. They draw on Vygotskyan frameworks that stress the interaction between language and cognition and highlight the social dimension of learning and the role of communication and participation. In particular, Vygotsky's proposal that all learning begins in interaction and that it is through social interaction through language that children develop the more formalized concepts of schooled subjects has been very influential.

These two prominent views; on the one hand, that students construct mathematics knowledge for themselves as they develop the mental representations that are necessary for doing mathematics, and the other that students are enculturated into mathematics through social and discursive activity (and of course this simplifies both positions), are currently being extended in approaches to research on mathematics education that call themselves *semiotic* (see, for example, the papers in Anderson, Sáenz-Ludlow, Zellweger, & Cifarelli, 2003 and Cobb, Yackel, & McClain, 2000). Semiotic approaches take different forms and draw on different epistemological orientations, but they recognize both language and mathematics as complex meaning-making systems. Semiotic approaches have in common the view that language is more than a tool for representation and communication. It is a tool for thinking and constructing knowledge. Many researchers who work from a semiotic perspective are also interested in interpersonal and developmental aspects of learning. Sfard (Sfard, Nesher, Streefland, Cobb, &

Mason, 1998; Sfard, 2001) for example, suggests that "all our thinking is essentially discursive," (Sfard et al., 1998, p. 50) and puts forward what she calls a *communication approach* to cognition (Sfard, 2001). McNamara (2003) characterizes a semiotic approach as understanding that "[k]nowledge is actively constructed by the cognizing subject, not passively received from the environment...[and] [c]oming to know is an adaptive process that organises one's experiential world... (p. 30). She suggests that it is necessary to adopt this understanding in order to see that language is not just a labelling process in mathematics, but that in fact language is the means for developing mathematical ideas.

Semiotic and information processing approaches are contrasted by Radford (2000), who analyzes students' thinking about algebra. From the information processing point of view, students are expected to extract meaning from the structure of the language as they manipulate symbols. But this point of view does not acknowledge the contribution of the students' experience and the social nature of the language and context for learning. Radford points out that students need to learn to reason in new ways in learning mathematics and that their prior knowledge and experience shape this in different ways. If we understand thinking as social practice in context, it is clear that students coming from different cultural contexts will be positioned in different ways to take up mathematics knowledge.

These differences can be acknowledged and made resources in mathematics classrooms if the focus is on meaning and if teachers are able to hear different perspectives. Sáenz-Ludlow (2003) argues that a semiotic approach can transcend arguments about whether it is the cognitive activity of the individual that is primary, with social interaction necessary but secondary, or whether knowledge is constructed as people interact. Her point is that cognitive activity and social interaction co-exist and co-act synergistically to support evolving understanding that involves both interpretation and the construction of mathematical meanings. She points out that when there is interpretation and construction there will also be self-expression, again highlighting the interpersonal aspects. She suggests that "…semiosis is a signifying process in which thought, language, and culture interweave to induce and sustain interpretation, construction, and expression of knowledge" (p. 260). She recommends "interpreting games" that enable the teacher to hear how students interpret mathematical meanings, using different wording to express the meanings in mathematical notation. Such interpretation, she suggests, enables movement from concrete contexts of doing mathematics to more abstract mathematical understanding. Students begin to understand the links among concepts through their own idiosyncratic symbols and formulations.

Linguistic tools, based on a social semiotic view of language, enable researchers to link expression with meaning in comprehensive and theoretically motivated ways so that language can be related to context in ways that show how meaning is made in the different activities of mathematics education. Sfard and Lavie (2005) point out that researchers get more reliable results when they analyze children's exact words, as analyzing actual utterances guards against interpretations of children's talk that are influenced by adult understanding. They use linguistic evidence to examine the way notions such as *comparison* are used by young children, analyzing how they apply the adjectives *bigger* and *smaller* or the adverbs *more* and *less* and show that young children's talk about numbers is not objectified as adults' is, and that the way the children talk about comparison shows that they are not using the same meaning systems as adults do. Sfard and Lavie point out that it is the use of language and engagement with concepts that provides the experiential contexts where children come to shape their meanings toward adult understanding.

Sfard & Lavie (2005) suggest that the path from everyday knowledge into the more specialized knowledge of formal mathematics can be thought of as the development of a specialized discourse that requires *objectification.* They illustrate how, in learning arithmetic, young children begin with ritualized participation in routines of counting and interacting with adults that then develop through discourse into more formalized knowledge. They recognize objectification when children begin using number words to signify entities that are not just concrete objects in the immediate context. For example, when children use number words as determiners (e.g., *three blocks*), they are not objectifying, whereas when they begin to use number words as nouns (e.g., *three is greater than four*), objectification is recognized.

The notion that close analysis of the forms of language itself is important for understanding the meanings that are being construed is formalized in the social semiotic approach of systemic functional linguistics (SFL), which provides a comprehensive framework for linking linguistic realization (the way the concepts are articulated in language) with the meaning that is thereby construed. SFL is based on the notion that meaning, and therefore also language, varies according to social and cultural context. It suggests that the choice of the grammatical form (*choice* in terms not of conscious selection but of unconscious use) varies according to the contexts in which speakers interact, and that analyzing language choices can reveal important differences in how content, role relationships, and information flow are constructed by different speakers in different contexts. From this perspective, analysis of the actual language used in mathematics teaching and learning reveals the knowledge that teachers make available in mathematics classrooms and how students take up new knowledge in the context of actually doing mathematics, as well as the way students are positioned and engaged as they learn (examples to illustrate this are presented below).

SFL provides an elaborated grammar of English that links meaning with the grammatical forms through which meaning is made (Halliday & Matthiessen, 2004) and enables exploration of the patterns of language through which meaning is constructed in interaction and in pedagogical texts. This social semiotic approach was introduced to mathematics educators in Halliday's early essay on the mathematics register (Halliday, 1978), often cited by researchers. Without a comprehensive understanding of the linguistic framework, however, researchers sometimes interpret the mathematics register as merely a set of words and phrases that are particular to mathematics.

Halliday defines *register* as "a set of meanings that is appropriate to a particular function of language, together with the words and structures which express these meanings. We can refer to a 'mathematics register', in the sense of the meanings that belong to the language of mathematics (the mathematical use of natural language, that is: not mathematics itself), and that a language must express if it is being used for mathematical purposes" (Halliday, 1978, p. 195). He stressed that this is not just a question of learning new words, but also of learning new "styles of meaning and modes of argument…and of combining existing elements into new combinations" (Halliday, 1978, p. 195-6). Halliday points out that counting, measuring, and other "everyday" ways of doing mathematics draw on "everyday" language, but that the kind of mathematics that students need to develop through schooling uses language in new ways to serve new functions. The notion of a mathematical register helps us see in language how mathematical knowledge is different from knowledge in other academic subjects and recognize the ways students need to be able to use language to effectively participate in the ways of knowing that are particular to mathematics.

Learning the language of a new discipline is part of learning the new discipline; the learning is not separate from the development of the language that constructs the new knowledge. As with all school learning, a key challenge in mathematics teaching is to help students move between everyday, informal ways of construing knowledge and the technical and academic ways that are necessary for advanced learning. Since students come to school with everyday language with which they have constructed their knowledge of the world, the school can build on that knowledge and language to move students toward new, more scientific and technical understandings through consciousness about and attention to the linguistic challenges that accompany the conceptual challenges of learning.

The work of Jay Lemke has been seminal in drawing on SFL theory to focus on the way mathematical knowledge is constructed. Lemke shows that the mathematics symbolic language developed out of natural language so that the two systems are integrally related, as "[t]he history of mathematical speaking and writing is a history of the gradual extension of the semantic reach of natural language into new domains of meaning" (Lemke, 2003, p. 217). Going beyond Pimm's metaphor that "mathematics is a language," Lemke shows in nuanced ways just how language and mathematics are related, describing how, as mathematics developed historically, it drew on and developed out of natural language, creating the mathematics symbolism and drawing on visual display in distinctive ways.

Lemke has been a major contributor to the evolution of a social semiotic view of mathematics, and in addition to his description of the evolution of mathematics language, he has made two other key contributions: 1) the development of the notion of *thematic patterns*, based on his analysis of language in science classrooms (Lemke, 1990), and 2) the description of the interaction of the multiple semiotic systems that work together in mathematics (Lemke, 2003). Thematic patterns are the patterns of meaning built up by teachers and students as particular topics are developed in mathematics classrooms. Lemke (1990) calls mastery of thematic patterns the most essential element in learning. He demonstrates that when students use a different thematic pattern than the teacher does in talking about a topic, they are also thinking about it in different ways. The different semantic combinations have different meanings, and in seeing the meanings the students construe, we can see what they are understanding and taking up from the teacher and where their understanding does not fit with the official discourse of mathematics. Lemke points out that "[i]n teaching…any subject, we do not want students to simply parrot back the *words*. We want them to be able to construct the essential *meaning*s in their own words, and in slightly different words as the situation may require. Fixed words are useless. Wordings must change flexibly to meet the needs of the argument, problem, use, or application of the moment. But they must express the same essential *meaning*s if they are to be scientifically acceptable and, in most cases, practically useful. This is what we mean when we say we want students to "understand concepts" (p. 91).

Lemke shows that we can explore the concepts students are developing by analyzing the thematic patterns that they use. Chapman (1995) applies these insights in her analysis of an algebra lesson, showing how a teacher builds up the notion of a *constant difference* in finding the slope of a line by starting with a visual representation, a pattern of dots. Chapman traces the thematic patterns in the teacher's development of the notion that *a pattern of dots generates numbers* that *lead to a rule*. The procedure for finding the rule involves completing an $x + y$ *table* and looking for the *common difference.* Over the evolution of the teacher's explanation the formulation *the numbers* is reformulated as *the table,* the *difference pattern* is reformulated as the *constant,* and the *constant* is said to *lead to* the *linear* pattern. Students need to be able to follow the semantic shifts that occur as an explanation like this evolves. Tracing the thematic patterns shows the conceptual complexity and the challenges students face in making the connections in the way the mathematics understanding requires. Analysis of thematic patterns offers a methodology for identifying key concepts, seeing how they are presented to students and how students take them up, providing a powerful tool for investigating students' learning.

SFL theory also enables researchers to take into account the multi-semiotic nature of mathematics. Lemke (2003) points out that mathematics discourse enables us to present "meanings that natural language has trouble articulating" (p. 214) through the interaction of natural language, mathematics symbolism, and visual display. Spatial-visual proportions and continuous covariation of discrete operations on continuous variables can be represented in mathematics discourse where natural language, mathematics symbolism, and visual representations "form a single unified system for meaning-making" (p. 215). Lemke points out that discrete (typological) meanings are the domain of natural language, where we focus on discrete processes and things, while continuous (topological) meanings are the domain of mathematics, where fractions and complex ratios are easier to construe through mathematics symbolism and in visual display than in natural language. But it is the three systems working together that make up mathematical reasoning.

SFL theory provides a means of understanding how semiotic choices work together so that students can be made aware of the different strategies used for construing meaning in language, symbolism and visual images. In his analysis of a physics classroom discussion, Lemke shows that "...it would not be possible to get a complete and correct meaning just from the verbal language in the activity, nor just from the mathematical expressions written and calculations performed, nor just from the visual diagrams, overheads, and chalkboard cues, nor just from the gestures and motor actions of the participants. It is only by cross-referring and integrating these thematically, by operating with them as if they were all component resources of a single semiotic system, that meanings actually get effectively made and shared in real life" (p. 229). He also suggests that "[t]oo much opportunity for gaining mathematical understanding and intuition, too much practice at learning how to use mathematical meaning in real situations, is lost if mathematics is not taught, particularly at the introductory level, as a co-equal partner with language and visual representation in the analysis of natural and social phenomena" (p. 231).

The linguistic features of the mathematics register have recently been more fully elaborated by O'Halloran (2005) in a comprehensive grammatical description of the register features of mathematical discourse. She describes the roles of each of the semiotic systems, pointing out that natural language plays a major role in contextualizing problems, explaining the sequence of activities that need to be undertaken, and discussing the implications of the results. "Language is often used to introduce, contextualize and describe the mathematics problem. The next step is typically the visualization of the problem in graphical or diagrammatic form. Finally the problem is solved using mathematical symbolism through a variety of approaches which include the recognition of patterns, the use of analogy, an examination of different cases, working backwards from a solution to arrive at the original data, establishing sub-goals for complex problems, indirect reasoning in the form of proof by contradiction, mathematical induction…and mathematical deduction using previously established results" (p. 94).

The mathematics symbolism developed out of and depends on natural language. "[M]athematical symbolism developed as a semiotic resource with a grammar through which meaning is *unambiguously* encoded in ways which involve *maximal economy* and *condensation*" (p. 97). The symbolism enables mathematical meanings to be presented in exact ways, but in doing so it condenses meaning into structures which are not found in natural language, and so requires specialized knowledge about how this semiotic resource works. At the same time, the condensation and economy are functional for making the kinds of meanings that mathematics enables. O'Halloran (2005) points out that "...mathematics is seen to deal with a limited semantic field in limited ways, but in doing so has the potential to solve problems which would be impossible to solve using other semiotic resources" (p. 24).

Building on Halliday's definition, O'Halloran shows how the mathematics register can be systematically analyzed and described with a theoretically grounded means of linking form and meaning. SFL theory recognizes three kinds of meanings that are simultaneously realized in all mathematics discourse: *ideational* meaning that is both *experiential* and *logical*, *interpersonal* meaning that positions interlocutors in particular ways, and *textual* meaning that organizes and presents information. Each of these kinds of meaning can be linked with the linguistic expressions that construe it. To simplify somewhat, *experiential* meaning is realized in verbs, nouns, prepositional phrases and adverbs; *logical* meaning is realized in conjunctions and connectors; *interpersonal* meaning is realized in clause mood (whether statements, questions, or commands) and modality (degrees of *likelihood, obligation, usuality,* etc.); and *textual* meaning is realized in the way information is organized and presented.

The central element of the clause, the basic unit of language, is the *process,* constructed in the verb or verb phrase. Each process may have associated *participants* (typically constructed in noun phrases) and *circumstances* (typically constructed in adverbs and prepositional phrases). The mathematics register draws on only a subset of the elements of the linguistic systems that characterize natural language to construe experiential meaning. SFL distinguishes six process types that occur in natural language (*material, mental, behavioral, verbal*, *relational,* and *existential.* O'Halloran demonstrates that mathematical discourse, mainly concerned with describing and manipulating relations, uses few *material, mental, behavioral,* or *verbal* processes; instead, *relational* processes (processes of *being* and *having*), and *existential* processes (processes of *existing;* e.g., *Given ....*) dominate. O'Halloran identifies a seventh process type, the *operative* process (processes of *addition, subtraction,* and other calculations), as an innovation of mathematics discourse. Operative processes and their participants enable the mathematical symbolism to reconfigure structure configurations in solving problems; so, for example, $s(t) = -16t^2 + 80t$ is a combination of 'components' ($s(t)$, 16, t, 80, +) in 'expressions' ($-16t^2$, $80t$) that have been configured into a 'clause' where, for example, the expression $-16t^2$ is a reconfiguration of operative processes and participants ($-16 \times t \times t$) (examples from O'Halloran, 2000). O'Halloran points out that the mathematical symbolism "is functionally organized to fulfill the goals of mathematics: to order, to model situations, to present patterns, to solve problems and to make predictions" (p. 108). The *participants* in mathematical processes typically are numbers and variables which function as general representations rather than specific entities. The *circumstances* are only those that describe relationships and not the full range of circumstances used in natural language. O'Halloran characterizes the experiential meanings typical of the mathematics register as an "expanded realm of meaning within a restricted experiential field" (p. 110). She points out that spatial and positional notation also realizes experiential meaning in forms not found in natural language, and that all of these features make maximal structural condensation is possible.

In terms of interpersonal meaning, O'Halloran shows that "[t]he objective and factual appearance of mathematics results from a combination of the restricted selections in the fields of experiential and interpersonal meaning in the mathematical symbolism...the textual strategies of condensation through which meaning is efficiently encoded, and the emphasis directed towards logical meaning" (pp. 113-114). The logical meaning depends on knowledge and principles that are often not made explicit in the mathematics text, adding to the challenges for students. The textual organization of mathematics is "highly formalized in order to facilitate the economical encoding of relations which permits immediate engagement with the experiential and logical meaning of the text" (p. 121). Again, here the use of spatiality is a key element, as the reading path is not necessarily linear, and mathematical discourse permits more ellipsis (expressions that are left unstated but assumed) than natural language. This textual organization also permits maximum condensation. All of these resources, then, meet the aims of encoding patterns of relations economically and exactly, and permitting reconfiguration of elements as needed as problems are solved. The complexities of the grammatical descriptions O'Halloran develops are too extensive to review further here, but as the research based on this work is described below, the approach will be better elucidated and some of the constructs elaborated.

O'Halloran (2005), like Lemke (1990), recommends that the nature of the language itself become a point of discussion with students and that they engage in analysis of the ways the different elements of mathematical discourse interact. She suggests that through this approach, "students may come to understand that language is a tool used to create order, and that…content is only one aspect of the order…. Equally important are the social relations which are enacted, the logical reasoning which takes place and the ways in which the message is organized and delivered. In addition,…students can appreciate that there are culturally specific ways in which language is used in different contexts. The understanding of those ways, and the interests served by such language selections, opens the way for a critical engagement with texts" (p. 200).

Summary

Mathematics education researchers have sometimes constructed a dichotomy between the *cognitive* and the *social,* with different researchers staking out territory in one or the other of these areas. A social semiotic perspective allows a synthesis of these views, where cognition is seen as situated activity that is socially constructed and enabled through language and interaction. What the social semiotics of systemic functional linguistics adds to the semiotic approaches more broadly is a language for talking about language that enables analysis of the meanings construed by the three semiotic systems used in mathematical discourse (natural language, mathematics symbolism, and visual display) in interaction with each other. The social semiotic perspective based in systemic functional linguistics recognizes that differences in wording and meaning have implications for the learning that is taking place and the forms of consciousness about mathematics that students develop. Work drawing on this perspective can help us understand the role that language and classroom interaction play in providing students with access to mathematics and can help us see where students in some classrooms may not be getting such access. SFL lets us look at the processes of teaching, learning, and doing mathematics by identifying the meanings that are made in the texts and interaction. Language is seen as *construing* meaning; actually making the meaning available through the realization in spoken or written language. The meanings that are construed are related to the contexts of use, and these are recognized at both local (context of situation) and more global (context of culture) levels. Using a lens that focuses on one or the other level, or both, provides insights into the meanings that are made.

Seeing language as constitutive of meaning allows us to analyze the language produced by students and teachers in classrooms, or the language of the textbooks and other explanations students work with, to recognize how differences in the way language is used to construct the content knowledge of mathematics and the way it is used to contextualize that knowledge and facilitate students' interaction with it. As we will see, a major tension teachers face is in finding middle ground between the knowledge and language students bring and the knowledge and language they need to develop in mathematics classrooms, and a means of talking explicitly about the language through which mathematics is constructed offers a way of responding to this tension. As is explored below, researchers have used SFL to show how students in classrooms of different kinds experience mathematics differently and are offered differential opportunities to get access to this powerful discourse. These insights enable us to recognize differences in the ways mathematics is construed in schools with students from different linguistic, social class, or ethnic backgrounds and to link the construals with opportunities to learn mathematics, providing us with ways to intervene and notions of what can be improved in mathematics education. This review will return to these points after considering the classroom pedagogical issues that research in mathematics education has raised and discussed.

Language in mathematics classrooms: Dilemmas and tensions

Studies of language in mathematics classrooms raise concerns that connect with the theoretical considerations discussed above. How is mathematics different from other school subjects? How can talk about language support students' mathematics learning? How explicit and precise should a teacher be in presenting mathematics concepts and how much everyday language and technical language should be used? How can teachers of diverse learners use language effectively in teaching mathematics? How does language affect assessment of students' mathematical knowledge? Research on these questions is reviewed here.

The two key subjects in school learning are language arts and mathematics, and some work on mathematics education has promoted strategies from language arts such as process writing and a more narrative approach as a way of making mathematics learning less formal or less alienating. Such practices have been critiqued, however (see discussion in Schleppegrell, 2007). Mathematics is different from language arts in the way it develops knowledge through relatively precise language and formal models (Bernstein, 1996), drawing on language that is different from the language that is typically used in language arts. Solomon & O'Neill (1998), for example, point out that mathematical argument achieves cohesion through logical rather than temporal order, different from narrative. They point out that "[w]hile it is right to value children's own knowledge and experience, it is necessary…to teach them about different literacy practices which have different functions…and cannot be generated simply from everyday practices which have very different functions" (p. 219). They describe the genres through which mathematical meanings are constituted and suggest that those genres should be a focus of instruction (see also Marks & Mousley's (1990) critique of process approaches to writing in the context of mathematics teaching). Attempts to make mathematics relate directly to students' experience and to eliminate or downplay the technicality of mathematics may also be problematic, since the technicality is functional for making the kinds of meanings that are relevant to constructing knowledge in mathematics. As Pimm (1987) points out, although many people talk about how precise mathematics language is, the precision is in the way language is used. The technical language has to be practiced and developed along with the mathematics concepts.

As work on classroom discourse has illuminated classroom interactional practices, it has become clear that a focus on discourse is not just a question of stimulating talk in classrooms. While most mathematics educators agree that it is important for students to use language to explore mathematical issues, the kind of exploration that is needed has to be particular to the kind of knowledge that mathematics constructs. Silver and Smith (1996) point out that classroom talk needs to ensure that the mathematics does not get lost, and that the discourse centers on "worthwhile tasks that engage students in thinking and reasoning about important mathematical ideas" (p. 24). If the discourse centers only on giving the right answer or on procedural issues, it does not model mathematical thinking and reasoning. Teachers co-construct mathematical explanations more effectively with some students than with others, providing different opportunities for students to develop the mathematics register (Forman, McCormick, & Donato, 1997). To effectively interact in ways that support mathematical development, teachers need to recognize what a student is trying to say and improve the student's ability to articulate it rather than seeing language merely as a way to assess students' knowledge (Laborde et al., 1990).

The notion of precision in the representation of concepts and clarity about what is being communicated has been a prominent theme in work on language and mathematics and has led to much discussion of the dilemmas teachers face in reconciling precision and clarity with the messy task of initiating students into the technicalities of mathematics discourse. Related to this is the role of everyday language in mathematics learning. Some mathematics concepts can be articulated in everyday language, but some require using language in new ways. For example, MacGregor (2002) found that students need to develop new ways of using language in order to structure mathematical concepts in the precise ways that are required for participation in advanced mathematics contexts. She illustrates this with an example of comparison as it was construed by the Australian students and pre-service teachers that she studied. A student explains *if Tina has twice as much money as George, then George has twice as less than Tina*. The student is using *twice as less* instead of *half as much,* but such usage does not construct the concepts of difference and comparison with the precision that is needed to enable further development of these concepts in higher mathematics. MacGregor suggests that "the grammatical form of a comparative expression plays a part in determining the form of the associated concept" and that "grammatical form…can reveal that a concept is vague, under-developed, unstable or incorrect" (p. 2). Teachers need to recognize when students are using such expressions and understand their value in the colloquial language of students' home communities, but also help students adopt the mathematical discourse that will enable them to participate in mathematics in the formal context of schooling. MacGregor found that "secondary students who described a relation between numbers in an informal, unclear or immature way were unable to relate it to a mathematical operation." There are many such forms that are common in everyday situations, and such everyday ways of using language are not adequate for constructing the precise and technical meanings that mathematics requires.

In the pedagogical context the tensions related to the notions of everyday vs. technical language and explicitness vs. implicitness are often talked about in terms of the language of 'dilemmas,' following Lampert (1985). Adler, for example, in the complex multilingual context of South African classrooms, has discussed three key dilemmas: the *dilemma of mediation* (Adler, 1997; 1999), the *dilemma of transparency* (Adler, 1998; 1999), and the *dilemma of code-switching* (Adler, 1998). Adler (1997)'s *dilemma of mediation* focuses attention on the need for teachers to listen to and validate the perspectives learners bring, while at the same time moving them from their own formulations of concepts into the formalized discourse of mathematics. In her case study of a South African classroom where the teacher uses both student grouping and teacher-student interaction, Adler shows how enabling students to work together, without monitoring by the teacher, enables a participatory classroom culture, but that the teacher's intervention and mediation "is essential to improving the substance of communication about mathematics and the development of scientific concepts" (p. 255). Adler suggests that the dilemma is "in shaping informal, expressive and sometimes incomplete and confusing language, while aiming toward the abstract and formal language of mathematics," pointing out that "a participatory-inquiry approach, and the possibilities it offers for learner activity and pupil-pupil interaction, can inadvertently constrain mediation of mathematical activity and access to mathematical concepts" (p. 236).

The teacher's oral language is a key means of linking between visual and symbolic representations, making the spoken language very powerful in classroom learning and suggesting that the way language and mathematics interact can become an explicit focus of attention in classrooms so that knowledge can be presented to students in explicit ways (Veel, 1999). But the notion of explicit teaching, and making the language transparent in its meaning, is not a straightforward issue. Adler (1998) demonstrates that explicit mathematics language teaching, focused on instructions and explanations, helped all learners, but the teachers she interviewed felt uncomfortable with all the talking they were doing. They felt that their attempts to make everything clear were sometimes distracting from the development of the mathematics concepts. This is an issue Adler (1998) calls the *dilemma of transparency*; whether teachers should step in to clarify concepts or not. Gorgorió and Planas (2001) report on classrooms where teachers working with immigrant students in Spain found simplified language of little help in communicating mathematical ideas. A simple change in vocabulary was seldom effective in clarifying concepts, and simplified forms of language sometimes obscured mathematics knowledge rather than clarifying it, although in a search for transparency, teachers still often tended to simplify. This research showed that problem-solving activities in linguistically homogeneous groups minimized some of the linguistic difficulties and Gorgorió and Planas call for more research "to clarify how mathematical language can be taught and to investigate the relationships between the 'language of the mathematics class', mathematical language, and the process of constructing mathematical knowledge" (p. 30).

These issues are especially crucial for teaching and learning in multilingual and multicultural classrooms. Schooled language is a challenge for all students, but there are particular groups of students for whom the challenges are greater than for others. English language learners and speakers of nonstandard varieties of English are two of these groups. Adler's *dilemma of code-switching,* referred to above, addresses some issues that come up in multilingual classrooms where teacher and students share a common language in addition to English. *Code-switching* refers to the practice of moving between languages in seamless ways, a practice common in bilingual and multilingual settings. Setati and Adler (2001) describe code-switching in a South African multilingual context where mathematics is taught in English, but the students and teachers also speak Setswana. The students in these classrooms hear English only at school, and although the teachers understand that talking together is a way of thinking together, using the students' own language is problematic when that language has not developed a mathematics register that engages with the kind of mathematics taught at school. This is the case for many languages in the world that are unwritten or not developed for higher education, such as Setswana, and also for nonstandard dialects of English such as African American vernacular. It does not denigrate students to say that their home language does not include an advanced mathematics register. For historical reasons, not every language or variety of language has evolved the specialized ways of meaning that construct the mathematics students are expected to learn in school. As Lemke and O'Halloran illustrate, mathematical discourse evolved over time out of natural language in the specialized contexts of mathematics use in specific cultural and linguistic contexts. Not every language variety has a register that has developed to do this kind of mathematics. Every language and dialect has the potential to develop such a mathematics register, but there is little social context for or political interest in investing in such development today for languages that are not already used for the kind of mathematics discourse done at school.

One way to address this is by recognizing the different registers that operate simultaneously in classrooms, so that the registers students bring can serve as resources in the development of new registers. In the South African classrooms she studies, Setati (2005) distinguishes four registers (she calls them *discourses*), referring to when the language is about procedural steps to solve a problem, when the focus is on reasons for calculating in particular ways, when the language builds the context of word problems, and when the language is used to regulate students. The teacher moves between English and Setswana for these different purposes. Setati points out that English is the language of assessment and so provides access to the social goods that come through learning mathematics, but at the same time perpetuates the hegemony of English in South Africa. Not providing access to English risks marginalization of the students at the same time that the teacher needs to use Setswana to manage much of what is going on. Setati suggests that teachers think about the different opportunities the registers offer for responding to and validating the language the students bring to the classroom on the one hand, while seizing opportunities to move the students in the direction of the more technical mathematics register when constructing mathematical knowledge. Working in contexts where students speak dialects or languages that do not have mathematics registers that can be used in classroom learning is a challenge, and research on this issue that recognizes the role of register and that can distinguish the use of different registers in the classroom is needed.

The role of the technical language is an issue recognized by research on English

Language Learners (ELLs) in U.S. contexts as well. In bilingual settings it is important that teachers know the mathematics and can use the technical register in both languages where two languages are used and both have a mathematics register (e.g., English/Spanish contexts). Khisty and Viego (1999) suggest using technical language with Latino bilingual students and focusing on reasoning rather than correct answers in responding to students. Moschkovich (1999; 2000; 2002) analyzes teacher-student interaction in classroom episodes with Latino students in California. She shows how these students use gesture and other modalities in bilingual conversations to make meaning and participate in mathematical discourses. She points out that manipulatives and pictures cannot be used without language and grapples with the issue of the movement between informal and technical language, suggesting 'revoicing' students' contributions to make them more mathematical, and not correcting students' language errors (Moschkovich, 1999). She points out that teachers and learners are not always talking about the same thing and critiques the typical approaches offered through generic ESL strategies because they have no mathematics content or guidance on how to concentrate on the mathematics content of discussions.

Lager (2006) also illustrates that generic ESL strategies are not enough. He points out that teachers know they need to create and use manipulatives, form cooperative groups, and increase their content knowledge, but they do not have rich knowledge about the role of language in facilitating these strategies or in making mathematics comprehensible and accessible to students. He reports on a study of ELLs and non-ELLs responding to a set of middle school algebra items about a linear pattern and shows how just one misunderstanding can lead to logical but incorrect solutions that then affect each subsequent item. He points out that "modeling problem situations requires translating from everyday language to algebraic expression...including the reorganization and reinterpretation of problem information" (p. 167).

It is also not practical to think that students can learn the language they need outside of mathematics classrooms and so be prepared, for example, by ESL teachers who themselves are not mathematics teachers, to deal with the language of mathematics. Barwell (2003, 2005a, 2005b) analyzed nine- and ten-year old ELLs interacting in mainstream classrooms in the UK as they wrote word problems, focusing on what they attended to in completing the task. He found that these students paid regular attention to mathematical structure, to the shape of the genre, and to written aspects of the work (spelling, punctuation, tenses), and points out that for the students, the content and language were not separate. He calls for "a more explicitly reflexive model of the relationship between content, language, and learning" (2005a; p. 206) that sees language and mathematics as jointly constructed and not separate. These studies reinforce the notion that the movement between the language that students bring and the new ways of using language they need to develop depends on teachers' abilities to recognize how best to construct mathematical meanings in language.

English language learners are a heterogeneous group and they do not all face the same challenges in school learning. One useful distinction to make is between those who have developed literacy and worked at grade level in their first language and those whose education has been interrupted, or who have never had an opportunity to develop literacy in their first languages. Students with literacy and grade level knowledge in their first languages will be able to move more quickly into the same content knowledge in English, as they will already have developed understanding of mathematics concepts in that language and will be aware of the mathematics register in their first language. In addition, students whose only opportunities to learn are in the classroom and whose lives outside of school do not offer them experience in talking about mathematics or engaging with mathematical meaning will have more difficulty and will need more experience in order to develop mathematics knowledge and control of mathematics meaning-making. Such students share many challenges with speakers of nonstandard dialects who do not encounter academic English outside of school.

Cultural differences related to language use also affect expectations about student-teacher interaction. For example, in the classrooms Gorgorió and Planas (2001) report on, students have difficulty communicating in Catalan, the language of instruction, even when they 'know' the language, because of cultural constraints on telling the teacher when they do not understand and because of other different communication norms. The teachers they studied also did not recognize that students could continue to have difficulty understanding mathematical discourse even when they seemed fluent in Catalan for other purposes.

Notions of universality and cognitive invariance across cultures are currently being critiqued, with accompanying interest in descriptions of the processes of learning and how they might differ across cultures (Sfard, 2001). Students may draw on different epistemologies and ways of knowing and doing mathematics that emerge from cultural differences (Selin and D'Ambrosio, 2000). Nunes, Schliemann, and Carraher (1993), for example, show how mathematics is used by fishermen, farmers, and carpenters in Brazil to solve problems informally, and argue for a more realistic mathematics in the classroom (see also Presmeg, 1998). But even when they build on everyday mathematical practices, classroom activities also need to offer opportunities for learning academic mathematics. Cultural and linguistic differences are potential resources in the classroom when teachers can recognize different voices and differences in views and make mathematical knowledge an object of reflection in ways that enable more students to participate in talking about and learning mathematics (Zack & Graves, 2001).

The issue of authenticity is often raised in the context of discussion about mathematics word problems. Word problems are contrived by teachers and curriculum writers and do not draw on everyday knowledge, even when they are intended to link to authentic contexts. We cannot simplistically use what seem to be everyday contexts and expect that doing so will help students learning the mathematics they need to succeed at school. In fact, making it appear that mathematics draws on everyday knowledge in situations where in fact it does not may even make the knowledge less accessible to struggling students. As we have seen, mathematics is a technical discourse, and while teachers need to connect to students and what they bring in understanding to the classroom, they also need to move students into the disciplinary ways of talking about and doing mathematics that will enable them to participate in advanced contexts of mathematics learning.

Researchers have investigated how the language of mathematics word problems influences children's comprehension and ability to solve the problems, but the difficulties are typically located in the situations and contexts that the problems present (e.g., Staub & Reusser, 1995). Gerofsky (1996) has described the structure of word problems from a narrative/genre perspective but we need to know more about the variables in word problems that affect students' comprehension, using frameworks that can link wording and meaning with students' difficulties. Some research related to ELLs performance in mathematics has focused on the wording in high stakes exams. Abedi & Lord (2001) analyzed the effect of modifying the language of released mathematics items from the National Assessment of Educational Progress (NAEP) 1992 to make meaning more accessible to struggling learners. They changed the language of word problems in several ways: shortening nominal expressions, making conditional relationships more explicit, changing complex question phrases to simple question words and passive voice to active, and replacing less familiar or less frequently used non-mathematics vocabulary with more common terms. They then interviewed 8th grade students who worked both the original and modified forms of the problems, asking them whether anything was confusing or easy about the problems and whether they would choose first to do the original or revised problem. Most students chose the revised versions and performed better with those versions. Low-performing mathematics students benefited more from the revisions than those in higher mathematics and algebra, ELLs benefited more than proficient speakers of English, and low socioeconomic status students benefited more than others. Brown (2005) has also contributed to this agenda, investigating literacy based performance assessments by third grade students in Maryland.

This is promising work that can contribute to the development of materials that provide better support for students who are moving from the everyday into the more technical language, and with a meaning-based grammar like that of SFL, we could engage in research that investigates variables such as how relationships between known and the unknown quantities are expressed, how explicit they are, what particular wording is chosen, and the features of distractors in multiple choice questions. Multiple choice questions comprise up to 70% or more of mathematics assessments (Veel, 1999), and we need to better understand to what extent mathematics questions test students' more general language skills as much as their understanding of mathematics. Such research could also help educators be clear about what we want students to learn and enhance the way standards are presented with more information about the language challenges and how teachers can address them.

Addressing the dilemmas

The dilemmas discussed above are persistent issues that are central to mathematics teaching and learning. Students' mathematical ideas are shaped in the interactions they participate in, requiring that teachers be responsive to what emerges as students engage in mathematical work, while at the same time being accountable to the instructional agenda. Herbst (2003) discusses this issue as a set of *tensions* that teachers need to be aware of concerning "the direction of students' activity, the representation of mathematical objects, and the elicitation of the conceptual actions that students need to invest" (p. 198). As students do new tasks, they are developing new knowledge through the task, and teachers have to pay attention to the final product or goal while taking account of students' conjectures and arguments on the way. Teachers also face tensions in how to represent mathematical objects, as the precision that is relevant to the mathematics is important, but so is some imprecision and vagueness in both teacher's and students' talk that allows students opportunities to think mathematically. Herbst's conceptualization of these dilemmas of mediation and transparency as tensions where no resolution is expected provides a practical way of thinking about the teaching process.

Teachers need to foreground everyday language in some contexts and technical language in others, and this has to do with the stage of development of the students' knowledge as well as the task that is currently being undertaken. In their conclusion to a special journal issue on language in mathematics education, Barwell, Leung, Morgan, & Street (2005) argue that "'fuzziness', ambiguity, multiplicity of meaning and exploratory discussion in everyday language should be recognized, not as failure to achieve a truly mathematical degree of precision, but as essential to making mathematical meanings and to learning mathematical concepts" (p. 144). They recognize the tension between precision and exploration and ambiguity and suggest that teachers need more explicit awareness of "the variety of forms mathematical communication may take, as well as a need for resources to support them in working with learners to develop a fuller understanding of the nature and role of mathematical language" (p. 145).

Raising questions about the language teachers use to talk about mathematics leads us toward study of classroom language with an eye to finding new ways of presenting mathematics knowledge in language at different levels, in the contexts of different topics, and in different social contexts. With knowledge about the linguistic features of the mathematics register, teachers are better equipped to make shifts between everyday and technical language in systematic and principled ways. For multicultural classrooms, a way of talking about mathematics that allows students to share perspectives and talk about their understanding offers an approach to diversity that can respect different perspectives but also provide a common way of negotiating understanding. We know that mathematics learning benefits from scaffolding through social interaction with a more expert interlocutor, and studies that describe effective interaction of this type are needed at all levels and in all instructional contexts.

Research from a functional linguistics perspective

This review has presented some theoretical perspectives that researchers have brought to the study of language and mathematics and has identified classroom issues that relate to language use in mathematics classrooms. Two key issues that a linguistic perspective can illuminate are highlighted here: the issue of how best to work with the multi-semiotic nature of the mathematics discourse itself and the issue of movement between everyday and formal ways of talking about mathematics. This section describes studies that illustrate how the linguistic tools and perspectives of SFL can help us understand and recognize how mathematics is construed in language and how students are positioned through language in mathematics classrooms.

Christie (1991; 2002) makes a useful distinction between the *content* or *instructional register* and the *regulative* or *pedagogical register* that projects the content. The technical mathematics discourse (the *content register*) is always embedded in and projected by the facilitative language of the classroom and text, the *pedagogical register*. The pedagogical register is the vehicle through which the content register is made available. Recognizing these registers in interaction with each other offers ways of negotiating the need to maintain an accurate representation of mathematics at the same time the language through which that knowledge is presented adapts to particular levels and contexts.

The mathematics content register itself has several dimensions. Research can explore the concepts that are presented, the logic of a text, the way a mathematics discourse positions students, and the textual organization of mathematics. In addition, research can explore how the mathematics content register is projected through the pedagogical register in different ways by teachers in different instructional contexts, with different kinds of learners, and at different stages and topics of mathematics instruction. With the linguistic tools that are now available, we are poised to be able to describe in greater detail, and with firm theoretical grounding, the ways meaning is made both in regulating classroom activity and in presenting the mathematics content, analyzing movement between everyday and technical ways of construing mathematics. In addition, recognition of the multi-semiotic nature of mathematics and the need to construct mathematics through movement among natural language, the mathematics symbolism, and visual representations opens new territory for research in the content register. Little has been done yet to explore how these systems work together, and research that explores this interaction in practice is needed. The studies described below provide some ways researchers have begun exploring these issues.

The nature of the knowledge being constructed in mathematics classrooms: the content register

Veel (1999) uses the SFL grammar to explore the features of the natural language used in mathematics and identifies distinctive ways that the elements of the clause appear in mathematics discourse. He points out that two types of relational processes, *attributive* and *identifying*, are pervasive in mathematics, and although they are often constructed by the same verb (forms of *is*), they present very different kinds of meaning. Attributive clauses classify objects and events, while identifying clauses introduce technical terms. An attributive process constructs information about membership in a class or part-whole relationship, as in: *A square is a quadrilateral* or *Three and four are factors of twelve.* An identifying process, on the other hand, constructs relationships of identity and equality, as in *A prime number is a number which can only be divided by one and itself* or *The mean, or average, score is the sum of the scores divided by the number of scores* (examples from Veel, 1999, p. 195). Identifying processes are particularly important because they can provide a bridge for students between technical and less technical construals of mathematics knowledge by enabling two formulations to be presented as equivalent (e.g., *Sides of the triangle that are in the same positions* ***are*** *<u>corresponding</u> sides of the triangles*). Relational processes are also a feature of the multiple choice questions that are often used to assess students' mathematics knowledge on standardized tests, as they ask *which of the following is correct/true/the best way,* etc. (Veel, 1999).

Clauses with the verbs *be* and *have* and other related verbs (*means, equals,* etc.) are challenging in their grammatical features. Students with first languages other than English may be accustomed to constructing relationships of attribution and identity in different ways than English. In Spanish, for example, the verb *is* has two different forms to construe the different meaning relationships. Veel is also interested in differences between the ways student use mathematical language and teacher/textbook use of mathematical language. He compares the lexical density (density of concepts) and the ratio of relational processes to non-relational processes in teachers', textbooks', and students' language and shows that there are gaps in the different formulations that are meaningful in terms of the knowledge constructed.

O'Halloran (2000) discusses the limitations of research on mathematics language that "largely centered around vocabulary, symbolism and isolated examples of specialist grammatical forms" (p. 395), and offers tools for analyzing discourse in ways that overcome these limitations. She analyzes the ways mathematics content is constructed by looking at what she calls the *nuclear configurations* (similar to Lemke's *thematic patterns*) in mathematics statements and oral discourse. By comparing the different ways teachers and students construct these nuclear configurations, she shows how the mathematics concepts are presented to and taken up by students. In addition, she analyzes the *reference chains* in the discourse to examine how concepts are introduced and developed and the complexity of the tracking that is needed to follow the argument being made. This entails looking at how mathematical participants in the discourse are split and recombined as a solution is developed, and how the logical relationships in the text are constructed. She points out that these logical relationships can be implicit or explicit and is interested in the ambiguity that is constructed in the movement between the oral language, the mathematics symbolism, and the visual display.

O'Halloran (1999) uses grammatical analysis to compare the way mathematical meaning is made in different classroom contexts. She shows the complexity of mathematical pedagogic discourse as it moves between oral and visual modes of presentation, drawing on symbols, diagrams, and language with dense texture. Long implicational chains of reasoning based on implicit mathematics results are often used to construct mathematical knowledge with register-specific technical terms, in very authoritative and sometimes alienating formations (O'Halloran, 1999).

Morgan (2004) illustrates how linguistic analysis can reveal differences in how mathematics is constructed for students at different levels by asking *"What is the nature of mathematics/mathematical objects/mathematical activity?"* and *"Where do power and authority lie?"* (p. 6). This research helps us understand how texts used for pedagogical purposes might better recognize the nature of mathematical knowledge and negotiate its authoritative nature. She is interested in how mathematics concepts are presented to students in the texts they read, pointing out that mathematics is often presented as a set of defined concepts that can be used to generate predictable results. This can lead to the kind of procedural pedagogy that presents mathematics as a process of learning word definitions and applying them.

Morgan is responding to mathematics standards in the UK that seem to focus only on vocabulary as a language issue and she shows how the language challenges students face go beyond word meaning. She analyzes definitions in the writing of mathematicians in academic journals and compares them with the way definitions are presented in textbooks at different levels. She shows that only textbooks present definitions as static; that in actual mathematics, definitions are presented as dynamic and evolving, open to decision-making by the mathematician. She illustrates this by comparing the way *agency* and other linguistic meanings are construed in the two kinds of texts. For example, she shows that where the mathematics research papers tend to present the authors as developing a definition (e.g., *we give a...definition*), the student texts, especially lower level texts, present definitions without ascribing agency (e.g., *X is called...*). She finds that higher level textbooks are more like the journal articles in allowing uncertainty and evolving meaning with a logical argument rather than absolute facts to be accepted. She suggests that ambiguity and multiplicity of meanings in classroom discussion are an important step in students' developing understanding and raises questions about the focus on vocabulary and defining terms that is common in mathematics standards and practice. She suggests that 'clear explanations' may not always be the most important focus and that the more advanced texts provide better models for students of the creativity and purposefulness of mathematical practice. She argues that the model that standards set up for teachers is both too restrictive in terms of its view of language and unreflective of actual classroom practice, where teachers have to move between the technical and the more everyday language and concepts of the students.

The clause-based grammatical analysis of Veel and the discourse-based analysis of Morgan, along with the research of O'Halloran, described above, are examples of how linguistic tools are available to study the 'content' of the mathematics language that is constructed in classrooms at different developmental levels and in different contexts and to explore the value of different ways of presenting mathematics meaning and talking with students about those meanings. Following O'Halloran's (2005) grammatical descriptions of the mathematics register, studies could compare the depth of embedding of the key configurations students are being asked to work with, look at how the oral language that presents the same concepts is structured, and analyze the role of other semiotic systems such as the visual display and the mathematics symbolism to explore their role in students' abilities to understand and respond to the problems. The logical relationships that are built up in texts of different types, and whether they are implicit or explicit, are also of interest.

Interacting with and positioning students: The pedagogical register

Another potentially fruitful line of research that can be pursued with linguistic tools is examination of the way the pedagogical register interacts with the content register in mathematics classrooms. This focus enables us to look at how students are positioned as they learn mathematics and the way they see themselves developing in the context of mathematics teaching. Research on the pedagogical register also can illuminate issues related to negotiating the dilemmas explored in this review in terms of how teachers can interact with students in ways that facilitate and support mathematics learning.

O'Halloran (2004) shows how linguistic tools can shed light on interpersonal relationships in the classroom and on how students are positioned as learners. Comparing differences in patterns of meaning in classrooms with students of different social classes and genders, she shows how the nature of the mathematical knowledge, the positioning of the students, and the opportunities to learn vary in relation to social class and gender variables, offering students different opportunities to engage with mathematics. She analyzes three year-10 mathematics lessons in Perth, Australia. Lesson One is taught to a class of male students at an elite school, Lesson Two is taught to female students at an elite school, and Lesson Three is taught to working class students at a public school. O'Halloran finds differences in classroom practices and the way the spoken language is used that relate to social class and gender and argues that these differences have implications for the content of the mathematics knowledge that is made available to students. She maps out the sequence of activities in each lesson, distinguishing between activities related to the mathematics content and activities that disrupt the focus on content, based on Lemke's (1990) notion of activity types. She provides visual representations of the three lessons that illustrate the "clear and progressive structure" of Lesson One in contrast with the disjointed movement between activities in Lessons Two and Three. More technical terms are used with the male elite students, and only in Lesson One is the language of the blackboard context-independent, so that it constructs a clear explanation that will make sense when copied into notebooks by the students. The lesson progresses more logically and with more internal coherence than the lessons in the other contexts. While Lesson One is focused on the mathematics content, in Lessons Two and Three the focus on content is disrupted as the teacher engages in disciplinary activities or asides. O'Halloran is able to show that less mathematics knowledge is therefore made available to students. Lesson Three uses the fewest technical terms, often moving away from the mathematics content because of disruptions due to students' behavior and the teacher's response.

O'Halloran also found that working-class and female students were "more consistently oriented towards interpersonal meaning at the expense of the mathematical content of the lesson" (p. 192). The teacher and the male students in the private elite school interacted with each other directly, without the covert discipline strategies (*can I just have your attention please*?) or use of sarcasm as a control mechanism (*I just like things to be clear*) that she found in the female and working class lessons. In the male elite school, the mathematics content was foregrounded, and the interpersonal interaction was stable. In the female and working class lessons, on the other hand, the mathematics content was often backgrounded for disciplinary reasons, and the interaction of teacher with students did not show the same level of mutual respect. O'Halloran's fine-grained linguistic analysis using SFL provides evidence to support these findings. She also shows that the findings are reflected in the performance of students from these schools on national exams, with the male students scoring higher than the female, who in turn score higher than the working class students. O'Halloran raises questions about how the pedagogical register of the mathematics classroom positions students in different ways, providing differential access to participation.

Morgan (2006) also considers aspects of the pedagogical register by interpreting how the meanings made in different language choices position students differently. She suggests key questions that can be asked about how mathematical knowledge is constructed and how learners are engaged with such knowledge and identifies the language features and grammatical analysis strategies that can be used to answer the questions. Seeing the language as evidence of the knowledge that is at stake as participants engage with mathematics, her goal is to interpret "the functions that these features fulfill for the participants in the mathematical practices…to gain understanding of the practices themselves" (p. 226). She shows how the language constructs students as actively *doing* in some cases and the mathematics as just *given* in others (e.g. the difference between *if you do this, X increases* and *here is the formula*). She argues that these differences in wording "construct different images of the objects of mathematics and the nature of mathematical activity. At the same time they claim different types of authority and construct different 'ideal' positions for their readers" (p. 236). She also points out that looking at the actual language used in mathematics teaching and learning can help researchers recognize teachers' and students' beliefs about the nature of mathematics without relying on self-reports or responses to explicit or implicit questioning outside the context of actually doing mathematics.

Morgan suggests, for example, that when textbooks use procedural discourse as their organizational strategy, they "may make students more likely to perceive mathematics as consisting of a set of procedures, and hence, perhaps, to find it more difficult to engage with relational or logical aspects of the subject" (p. 228). Texts that obscure human agency "may contribute to difficulties for some students in seeing themselves as potential mathematicians" (p. 228). She illustrates this by comparing the texts written by two students in response to a problem, looking at how they represent mathematical objects, the processes they are involved in, and who is acting in the processes. By observing the patterns constructed by the students, she is able to show that one student draws "primarily on a discourse of investigation, oriented to value exploration of interesting mathematics," while the other "draws strongly on an assessment discourse, displaying the 'answers' valued within that discourse" (p. 236). By looking at the text as a whole, she can track changes in it as it progresses and is negotiated and show how meaning evolves. Morgan calls for more such analysis to show how texts construct different images of mathematics and how those are read by students, and how students adopt and reproduce the images of mathematics constructed by their teachers. A linguistic analysis can also show differences in teaching styles and student participation, the effect of resistance by students on teachers' evaluation of them, and how the social positioning of students in the classroom constructs their identities as effective or less effective mathematics students. Morgan suggests that knowledge about how various responses are evaluated is especially important in the current high stakes testing environment.

Research can also show how classroom practices in mathematics can lead to student alienation. In interviews with 12 first year university students, Solomon (2006) found that they did not see themselves as potential members of the mathematics discourse community, even when they were majoring in mathematics and interested in the subject. She blames this on a mode of teaching that focuses on procedures and rote learning rather than negotiation of meaning and development of understanding through participation. She suggests that "learners...often are excluded from the negotiation of meaning...developing instead an identity of non-participation and marginalisation. Their lack of ownership generates and is generated by compliance with authority and an emphasis on following pre-set procedures" (p. 376-7). She points out that while professional mathematics involves procedures such as "*intuition, trial, error, speculation,* *conjecture, proof"* undergraduate teaching of mathematics stresses "*definition, theorem, proof,*" and as a consequence, students see mathematics learning as rote learning, and see proof as instrumental and performance-related. As she summarizes her findings, "[a] general theme of certainty in mathematics emerged, coupled with an emphasis on the necessity of learning rules, reproducing solutions and working at speed to get correct answers" (p. 382). The students assumed "certainty, irrelevance, rule-boundedness and lack of creativity potential in pure mathematics" (p. 383) and were "largely unaware of the existence of a mathematics community of practice which might have negotiable rules of communication and validation" (p. 384). She suggests that the way mathematics is taught means that students are unlikely to be able to develop an identity as a mathematician, and that students need to "make the transition from a performance-oriented and individualistic view of mathematics...to a view...which emphasises construction, communication and community" (p. 391).

These studies suggest promising directions for research on the pedagogical register and student positioning in mathematics classrooms that may help untangle issues related to the different ways students encounter mathematics in today's multicultural classrooms and help teachers address the tensions they confront as they negotiate language and mathematics learning. By identifying features of the pedagogical register that facilitate the development of students' understanding, the role of language in talking about mathematical meaning can be better understood, and ways for teachers and students to talk about mathematics at different developmental levels and in different topics can be elucidated.

Intertextuality and the multi-semiotic nature of mathematics

Two other key issues also need further study in mathematics teaching and learning. One is movement between and among texts of different types, spoken and written, in classroom instruction, as students listen to the teacher, work with textbooks, and interact with the blackboard and other visual representations. Related to this is the need for more research on the role of the multi-semiotic nature of the mathematics content register in learning and teaching. The movement between and among texts of different types has been addressed in some research on intertextuality in mathematics. Studies have compared the language used by teachers and students or by students and textbooks in talking about the same concepts (Chapman, 1995; Veel, 1999), and these studies find that the students do not readily take up the language in the same ways that the teacher or the textbook use it, with implications for the learning that is taking place. Less work has been done on the multi-semiotic nature of mathematical discourse.

Huang, Normandia, & Greer (2005), for example, following a line of research based on SFL and Mohan (1986), analyze the knowledge structures that appear in teacher and in student discourse in a mathematics class where the teacher expected technical language from the students and encouraged them to talk about mathematics. They report that students, working in groups, "...could easily describe an equation or a graph, sequentially tell about procedures they have followed to solve a function, and suggest a method or solution. However, whenever students were pushed to reference relevant concepts or principles, explain a method used, or justify a decision made for either a method or solution, they frequently seemed to hesitate or to appear less capable" (Huang et al., 2005, p. 44). Huang et al. found that knowledge structures such as *classification, principles*, and *evaluation* were only used by the teacher. Students used only *description, sequence*, and *choice* knowledge structures, even when pushed by the teacher. The authors suggest that students need explicit instruction in articulating principles to move them beyond the practical aspects of mathematics knowledge in their discussion. They recommend that students be asked to "talk their way into habits of expressing higher-level knowledge structures" (Huang et al., 2005, pp. 44-45), and that teachers integrate thinking and talking at all levels. Borasi and Siegel (2000) also make connections between reading and talk and drawing in geometry and statistics and offer strategies for making links across modalities.

Chapman's work (1995; 2003a; 2003b), based on an ethnographic study of a year nine mathematics class in Australia, offers a rich model of how intertextuality can be studied. She is interested in language use by students and teachers as well as the role of texts, diagrams and other visuals interacting with the mathematics symbolism. She shows how knowledge is built up as students interact with spoken and written "texts" over the course of a unit on functions. (The discussion of thematic formations, above, draws from this work.) With a social semiotic perspective influenced by Lemke and SFL, she analyzes the semantic and thematic patterns and genre structures of the mathematics class, showing how the different texts in a unit of study offer opportunities for students to learn intertextually, and that in fact such learning is necessary, as no one element captures the entirety of the concepts that the unit develops. She shows that the thematic formations that construct the knowledge evolve through different patterns of expression, each pattern adding to the common pattern of expression that the teacher and students are building up.

Chapman (1995) shows how understanding is built up over the course of the whole unit as she analyzes the ways the teacher and students interact in whole group sequences, looks at the thematic patterns in teacher talk, analyzes the students' interaction in small groups, and looks at how the textbook represents the same thematic patterns. She shows that the teacher focuses students on using appropriate language to construct concepts and that she contextualizes the textbook language through spoken language to help students understand the relationships in the textbook definitions. Chapman also shows that the definitions in the textbook assume students already understand the relationships among what is defined. She illustrates that when the students work out a problem together in groups, some construct the concepts in ways that are different from the thematic patterns of the teacher and the textbook, sometimes leading to miscommunication as students use different thematic formations to talk about the mathematics content. Chapman suggests that the intertextual links can be highlighted for students if the teacher makes explicit reference to the key thematic patterns and relations, using ones that are familiar in making connections with other texts and other contexts, so that the thematic items are naturalized into the language of the classroom.

This work shows how mathematics concepts are developed through engagement with and talk about texts of different types, including homework, blackboard diagrams, the textbook, teachers-student interaction, and students' interaction with each other. It illustrates the role the teacher's language plays, showing how the teacher's language only makes sense in relation to the other texts the class engages with. Chapman also shows that mathematics texts require a different kind of reading than texts in other subjects, as the meaning relations do not unfold sequentially. They overlap, are repeated, and reinforce each other as they build new frameworks of understanding. Chapman urges that teachers explicitly reference the thematic patterns under construction as they transform nonmathematical expressions into mathematical ones.

In other work, Chapman shows that developing the mathematical discourse is a developmental process that evolves as students gain experience with new mathematical knowledge. Chapman (2003b) stresses that meaning is always produced in context and cannot be separated from social action, and that "ways of speaking that are appropriate to a subject-area are developed as part of the social practices of classroom interaction" (p. 133). Her notion of social context draws on Lemke in pointing out its three dimensions; the 'content' context, or what is being talked about; the sequential context, related to what has come before and what will come after, and the context of what has been selected to highlight in each instance, in contrast to what other elements might have been selected and highlighted. She suggests that these three angles on mathematics content helps us see where teachers are using everyday language in 'metaphoric' ways to represent mathematics. When she analyzes the relevance of the metaphoric content to the problem at hand, she finds that the more metaphoric ways of representing mathematics do not always represent the mathematics content accurately, as it is in the mathematics language itself that the mathematical meaning is made.

We need more research that compares texts of different types, reveals problems with different wording, and analyzes representations of mathematics at different levels and in different topics. Most needed is research that shows effective ways of negotiating the multi-semiotic nature of mathematics. We need to understand how teachers 'translate' across the different semiotics, identify where ambiguity occurs and its value, and develop more explicit descriptions of how language works across modalities. We need to know more about the language patterns and structures used in talking about visual elements in mathematics classrooms and the natural language used to interpret the mathematics symbols.

Summary

A functional linguistics perspective offers new ways of analyzing language to explore both how mathematics knowledge is constructed and how teachers initiate students into that knowledge in different contexts and at different levels. Studies that consider the actual language used in mathematics classrooms from a multi-semiotic perspective that recognizes the roles of language, mathematics symbolism, and visual display in constructing mathematical meaning can illuminate the issues this review has raised about movement between everyday and technical ways of making mathematical meaning and how best to engage students in diverse contexts. Analysis of the texts used for instruction and the ways teachers and students use and respond to those texts can identify challenges in the language and suggest ways to enable more effective presentation of content. Through comprehensive investigation of how technical meanings are made, the language structures that enable meaning, forms of interaction in the classroom, description of spoken and written genres, and the ways multisemiotic meaning-making works in mathematics teaching and learning, the language resources that are most relevant in constructing different mathematics topics at different levels can be identified and made explicit.

The role of teacher education

Mathematics teachers need more than knowledge about mathematics. Ball and Bass (2003) argue that knowledge about how to link content and pedagogy, what they call *mathematical knowledge for teaching,* is key to preparing effective teachers. Teachers need to know how to respond to children who offer alternative solutions, how to make mathematical ideas available to students, how to attend to, interpret, and handle students' oral and written productions; how to give and evaluate mathematical explanations and justifications, and how to establish and manage the discourse and collectivity of the class for mathematics learning (Ball & Bass, 2003).

Because teachers need to do interpretive work in developing and responding to students' learning in mathematics classrooms, Ball, Hill, and Bass (2005) see the teaching of mathematics as a form of mathematical work that requires "a special sort of sensitivity to the need for precision in mathematics…that language and ideas be meticulously specified so that mathematical problem solving is not unnecessarily impeded by ambiguities of meaning and interpretation" (p. 8). They report that they "have seen students struggle over language, where terms were incompletely or inconsistently defined, and we have seen discussions which run aground because mathematical reasoning is limited by a lack of established knowledge foundational to the point at hand" (p. 12), and suggest that "...an emergent theme in our research is the centrality of mathematical language...teachers must constantly make judgments about how to define terms and whether to permit informal language...When might imprecise or ambiguous language be pedagogically preferable and when might it threaten the development of correct understanding?" (p. 21).

This suggests that teaching mathematics is also a form of *linguistic work*. New registers evolve for students along with the development of their mathematics knowledge. As they do mathematics, they come to understand it in new ways. The challenge is that students have to use the mathematics register while learning the register. Teachers need to understand mathematics concepts well enough to be able to talk about them in both less and more technical ways, unpacking compressed conceptions or formal definitions of the technical language into the more informal, everyday language for student learning, but then compressing and repackaging it again in technical ways in order for the learning to build more complex concepts and move into new, more abstract domains. It is the movement back and forth that allows students to make connections across mathematical topics and for teachers to build links and establish coherence as learning develops.

Professional development can be a powerful venue for examining and disseminating new understanding of the relationship between language and mathematics so that we can better prepare teachers to deal with the tensions around language and content, raising teachers' awareness about language, giving them ways of explicitly talking about language features and modeling ways of responding to students' alternative explanations and questions. In the context of teacher development, a focus on language can help address the difference between questions like *do you know mathematics?* and *can you talk about mathematics?,* providing teachers with ways of engaging students in talk about the knowledge they are developing in ways that maintain the technicality and structure necessary for the concepts to be effectively engaged with.

The linguistic framework suggests ways of responding to the dilemmas identified by researchers by enabling talk about meaning: both talk about *what it means* and *how it means,* providing the potential for recognizing how meaning is constructed in mathematics and in mathematics classrooms. The SFL metalanguage (a language about language) can help negotiate the tensions, making the forms through which mathematics knowledge is presented explicit and providing a valuable tool that can be developed and studied in various contexts of use.

We know from research on school facilities and teacher quality that privileged students have more educational resources and achieve better outcomes than less privileged students. Understanding the linguistic challenges of mathematics may offer new ways of improving instruction for all students, despite these structural obstacles. The studies reviewed here illustrate the variety and richness of the insights available through functional linguistics tools, with methodological possibilities for research ranging from ethnographic studies to grammatical analyses and discourse analysis. More such research can enhance our understanding of the role language plays in mathematics education to help better understand what constitutes the mathematical knowledge for teaching that is most important.

Recommendations of this review

Research on classroom talk that shows effective ways of building on students' everyday language and developing their academic language can help us understand which areas of language are most readily addressed by teachers explicitly and what the outcomes are for students whose teachers are able to do this. A key issue is the complexity of using the natural language, mathematics symbolism, and visual display that make up the multi-semiotic mathematical discourse. How explicit should a teacher be, and how much everyday language and technical language should be used? Related to this is the issue of how to move students into a field that objectifies concepts and presents them in dense language that can be difficult to engage with. Should the students be given the formal definitions that present the knowledge mathematicians have developed, or should students be taken through a process that moves from less precise to more precise construction of knowledge? And how should teachers address the alienating quality of mathematical discourse and its authoritative constructions?

We need studies that illuminate the features of effective pedagogical and mathematics content registers in order to develop the metalanguage (language about language) that is useful for teachers and students at different levels and on different mathematics topics. Having a metalanguage that effectively supports students' development of mathematics concepts in different learning contexts can help teachers move beyond the everyday/technical dichotomy and start with the language students bring to school, but then move them into the ways of using language that effectively construct the mathematical knowledge they need to develop.

We are also need to analyze more fully the multi-semiotic nature of the construction of mathematics knowledge, and investigate how effective teachers shift among the different modes and semiotic systems as they negotiate movement between oral language, mathematics symbolism, and visual display. This needs to be studied in different contexts and at different levels, with various mathematics topics. Research that describes teachers' language and compares teachers' and students' ways of talking about mathematics can help us better understand the gap that has been seen between student use of mathematical language and teacher/textbook use of mathematical language, and where the developmental gap can be bridged. Studies of different kinds of texts, different ways of interacting in classrooms, and pathways that students' mathematical development takes can also be undertaken using linguistic tools that enable analysis of meaning-making in mathematics.

We need to look at different kinds of classrooms, situated in different social formations, to compare how students are positioned through the mathematics discourse and how teacher-student interaction varies across settings and the implications of this for teaching and learning. We need to understand differences in linguistic behavior among different language users and social contexts in the mathematics classroom and to compare teacher use of language in different contexts. Diverse contexts of research are necessary, as we cannot expect that one way of interacting with students will be effective with all. Research needs to include second language learners and speakers of non-standard dialects in a range of social contexts as well. It would be especially valuable to focus on settings where teachers have been successful to see how successful teachers use language in mathematics teaching in different learning contexts.

More research is needed that takes a developmental approach and shows how students develop knowledge over time. We need rich studies of how language and ways of talking about mathematics evolve over a unit of study, focusing on more than brief interactional episodes and fragments of dialogue. We need research that describes developmental paths into K-12 mathematics at different grades and on different topics for students in different contexts and with different backgrounds. Such research can also investigate how a metalanguage that enables connections to be made between language and mathematics meanings can support students' learning.

To make changes in the ways teachers use and think about language in mathematics classrooms, we need research that investigates the kind of understanding about language that it is possible for teachers to take up at different points in their development. Rich studies of mathematics teacher preparation where knowledge about mathematical discourse is made available to teachers and implemented in classrooms at different levels and in different contexts can provide information about the ways of using language that are most effective in helping students learn mathematics. Such studies should follow teachers' development and measure their performance. This is a crucial area for research and the new linguistic approaches have much to offer.

Semiotic approaches to mathematics education that recognize that knowledge is constructed in language and that mathematics is especially challenging in its multi-modal construal have much to offer researchers. Social semiotic approaches that recognize the different ways students from different cultures, language backgrounds, and social contexts use language in different ways offer especially rich opportunities to develop new approaches to mathematics education that recognize and value different ways of learning and using language. The dilemmas and challenges are clear and the methodological tools are available. A robust program of research that approaches these issues in systematic and deep ways can make major contributions to the field of mathematics education through study of language use.

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Note

[i] I come to this review as a linguist, bringing an applied linguistics perspective and a particular lens through which I view the work reported on here. The sources through which I identified relevant work reflect my disciplinary leanings and of course are not comprehensive of the field. Much more has been written on the topic of mathematics and language than it is possible to report here, but I have tried to be representative in discussing work that has made contributions to our understanding, with a particular focus on illuminating the contributions of linguists and applied linguists so that their work is better understood in the mathematics education community. My interest in the topic comes out of my interest in language in all school subjects. The goals in my research are to explicate the linguistic challenges of school subjects more generally (Schleppegrell, 2001, 2004), and my previous work has spoken to issues in language arts (Schleppegrell, 1996, 2003; Schleppegrell & Colombi, 1997), science (Schleppegrell, 1997, 1998, 2002), history (Achugar & Schleppegrell, 2005; Schleppegrell & Achugar, 2003; Schleppegrell, Achugar, & Oteíza, 2004; Schleppegrell & de Oliveira, 2006), and, more recently, mathematics (Herbel-Eisenmann &

Schleppegrell, forthcoming; Schleppegrell, 2007). My goal here is to identify key themes and major ideas in discussions of mathematics and language, focusing on central challenges and opportunities for work that would advance the field of mathematics education through a better understanding of the relationship between language and mathematics as it presents itself in the mathematics classroom. Understanding the ways researchers have seen the relationship between mathematics and language and identifying what is known about the particular features of the language of mathematics can help us formulate a research agenda that leads to greater understanding of the linguistic challenges of constructing mathematics knowledge and better preparation of teachers for engaging all students in learning mathematics.

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