Language(s) and learning mathematics: Resources, challenges, and issues for research

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1. Introduction

Integrating language into research on mathematics learning is an important goal for both practical and theoretical reasons. This integration is crucial for improving mathematics learning and teaching for students who are bilingual, multilingual, or learning English. This integration is also relevant to theory: research with learners who use more than one language can make language more visible than it might seem in monolingual situations, providing a window on the role of language in learning mathematics. In this essay I describe resources, challenges, and methodological issues to consider when designing research on the topic of language and mathematics learning. The first section of the essay focuses on the theoretical notions about language that are provided by different sets of research literature and that researchers should keep in mind if they address language. The second section describes several challenges to designing studies that are theoretically framed and methodologically sound. The third section focuses on the theory and methods that are relevant for data collection and interpretation.

Addressing the relationship between language and mathematics learning presents several challenges. The most significant challenge is that research examining language and mathematics learning must be grounded not only in current theoretical perspectives of mathematics cognition and learning, but also in current views of language, classroom discourse, bilingualism, and second language acquisition. Becoming familiar with two sets of research literature may seem like a daunting task. The first section of this essay describes resources currently available to design research on the topic of language and learning mathematics.

A second challenge in this endeavor arises because we all regularly participate in using language and, thus, we have developed intuitions about language based on our personal experience. Our personal experiences with language are couched in complex social, political, and historical contexts and our intuitions may have developed into language attitudes. Our intuitions about language may at times be in direct contradiction with empirical research on how people acquire language or use two languages. To address these contradictions, in the second section of the essay I describe common pitfalls to avoid when considering language in mathematics learning.

In the last section of the essay I describe important methodological issues to consider when designing research that addresses language(s) and mathematics learning, in particular when working with bilingual populations. This section focuses on issues to consider for data collection, transcription, and translation.

A few words about the term 'language.' Many commentaries on the role of academic language in teaching practice reduce the meaning of the term 'language' to single words and the proper use of grammar (for an example, see Cavanagh, 2005). In contrast, work on the language of specific disciplines provides a more complex view of mathematical language as not only specialized vocabulary (new words and new meanings for familiar words) but also as extended discourse that includes syntax and organization (Crowhurst, 1994), the mathematics register (Halliday, 1978), and Discourse practices (Moschkovich, 2007b). Theoretical positions in the research literature in mathematics education range from asserting that 'mathematics is a universal language,' to claiming that 'mathematics is a language,' to describing how "mathematical language is a problem.' Rather than joining in these arguments to consider whether mathematics is 'a' language or reducing 'language' to single words, I use a sociolinguistic framework to frame this essay. From this theoretical perspective, 'language' is a socio-cultural-historical activity, not a thing that can either be mathematical or not, universal or not. I use the phrase 'the language of mathematics' not to mean a list of vocabulary words or grammar rules but the communicative competence necessary and sufficient for competent participation in mathematical Discourse practices. I sometimes use the term 'language(s)' to remind us that there is no pure unadulterated language and that all language is hybrid.

2. Resources for designing research on language and mathematics learning

There are several sets of literature that can provide important theoretical notions when designing research studies on language and learning mathematics. Scholarly literature from several different fields, such as psycholinguistics studies on language switching during arithmetic calculations, sociolinguistics studies on code switching, research on classroom discourse in monolingual mathematics and science classrooms, and international research in multilingual classrooms, can contribute to designing further research[i].

By necessity, researchers in mathematics education who address issues of language have used work from fields outside of mathematics education to inform research on the relationship between language and mathematics cognition and learning. Researchers in mathematics education have used work from other disciplines to examine the role of language in mathematics learning in bilingual classrooms in the USA (Khisty, 1995, 2001; Khisty and Chval, 2002; Moschkovich 1999, 2002) and in multilingual classrooms in South Africa (Adler, 2001; Setati, 1998, 2003; Setati & Adler, 2001)[ii]. Work outside of mathematics education has contributed theoretical frameworks for studying discourse in general, methodologies (e.g. Gee, 1996), concepts such as registers (Halliday, 1978) and Discourses (Gee, 1996), and empirical work on classroom discourse (for example Cazden, 1986; Mehan, 1979). Work in second language acquisition, bilingualism, and bi-literacy (for example Bialystok, 2001; Hakuta & Cancino, 1977; Valdés-Fallis, 1978, 1979; Zentella, 1997) has also provided theoretical frameworks, concepts, and empirical results that are relevant to this research endeavor. This work has provided crucial concepts necessary for studying the role of language in mathematics learning, for example definitions of code switching (Auer, 1984; Gumperz, 1982; Zentella, 1981), distinctions among different types of code switching (Hamers and Blanc, 2000; Sanchez, 1994, Torres, 1997, Zentella, 1981), and the concept of hybridity (Gutierrez, Baquedano-Lopez, & Alvarez, 2001).

Borrowing concepts from other disciplines is a productive approach for integrating language into the study of mathematics learning. For example, the distinction between "national" languages----such as Spanish, English, or Haitian Creole---and "social" languages---such as mathematical or academic discourses--- is useful in clarifying what we mean when we use the term "language." While concepts and theories from other disciplines provide essential resources, borrowing concepts also presents challenges. There is danger in borrowing concepts and leaving behind the intellectual tradition that gives a concept meaning. Those of us whose expertise lies in mathematics education must remember that notions such as 'language,' 'culture,' and 'bilingualism' are as complex as any theoretical construct in our own field: these terms have contested meanings, long histories, and are the topics of heated debates in the fields of anthropology and linguistics.

In considering what work might be relevant to research on language and mathematics learning, it is important to distinguish between psycholinguistics and sociolinguistics because these two perspectives differ in how they explain and examine language. While sociolinguistics stresses the social nature of language and its use in varying contexts, psycholinguistics studies have been limited to an individual view of performance in experimental settings. From a sociolinguistic perspective, psycholinguistics experiments provide only limited knowledge about how people use language:

> The speaker's competence is multifaceted: How a person uses language will depend on what is understood to be appropriate in a given social setting, and as such, linguistic knowledge is situated not in the individual psyche but in a group's collective linguistic norms. (Hakuta & McLaughlin, 1996)

"Bilingualism" is one example of a concept that has different meanings depending on the theoretical perspective one uses to frame it. The two perspectives described above see bilingualism differently. A researcher working from a psycholinguistic perspective might define a 'bilingual' as any individual who is in some way proficient in more than one language. This definition might include a native English speaker who has learned a second language in school with some level of proficiency but does not participate in a bilingual community. In contrast, a researcher working from a sociolinguistic perspective might define a 'bilingual' as someone who participates in multiple language communities and is "the product of a specific linguistic community that uses one of its languages for certain functions and the other for other functions or situations" (Valdés-Fallis, 1978). This definition defines bilingualism not only as individual but also as a social and cultural phenomenon that involves participation in language practices and communities.

An important resource for research addressing language and learning mathematics is research carried out in geographic settings with student populations other than the target population for a particular research study. For example, researchers have studied language, bilingualism, and mathematics learning in Australia (for examples see Clarkson, 1991; Ellerton & Clements, 1991), Papua New Guinea (for examples see Clarkson, 1991; Clarkson & Galbraith, 1992; Dawe, 1983; Jones, 1982; Souviney, 1983), and in South African multilingual classrooms (for example Adler, 2001; Setati, 1998, 2003). This work can be an important resource for research with other student populations, as long as researchers note differences among settings that might be relevant to issues of language and learning mathematics for the student population for a particular research study.

What might be the relevance of work from Australia, Papua New Guinea, and the U.K. for Latino mathematics learners in the U.S.? What are the historical, political, and linguistic differences between the U.S. and South Africa that one should consider when using research from these two settings? Before applying research from Australia, PNG, or the U.K to U.S. settings and student populations, researchers should carefully consider relevant differences among settings, students, languages, and communities. One difference is that the U.S. Latino population of school age children can be largely described as bilingual in Spanish or as monolingual English speakers[iii]. In contrast, the majority of students (as well as teachers) in South African classrooms speak multiple indigenous languages at home. Another contrasting example is Pakistan, where the language of schooling is usually not spoken at home but reserved for activities related to school or government related activities. Barwell (2003) provides some useful distinctions among different language settings, using the terms *monopolist*, *pluralist*, and *globalist*. In monopolist classrooms all teaching and learning takes place in one dominant language; in pluralist classrooms, several languages used in the local community are also used for teaching and learning; in globalist classrooms teaching and learning are conducted in an internationally used language that is not used in the surrounding community.

Another difference to consider across settings and national languages is the nature of the mathematics register in students' first language. For example, individual mathematical terms exist in Spanish. Since university level courses and texts have existed in Spanish for centuries, the mathematics register in Spanish can be used to express many types of mathematical ideas, from everyday to advanced academic mathematics. This may not be the case for the home languages of students in other settings such as South Africa (Setati and Adler, 2001) or in the case of Australian Aboriginal languages (Roberts, 1998; Barton, Fairhall, & Trinick, 1998). These differences, however, should not be construed as a reflection of differences in learner's abilities to reason mathematically or to express mathematical ideas. Nor should we assume that there is a hierarchical relationship among languages that have different ways available for express school mathematical ideas. Instead, an ethno-mathematical perspective expands the kinds of activities considered mathematical beyond the mathematics found in textbooks or learned in schools (Bishop, 1986; D'Ambrosio, 1991; Nunez, Schliemann, and Carraher; 1993). This perspective emphasizes that mathematical activity is not a unitary category but is manifested in different ways in different settings, that all cultural groups generate mathematical ideas, and that "Western mathematics may be only one mathematics among many (Bishop, 1986)." Taking an ethno-mathematical stance means that no mathematical activity is seen as a deviant, inmate, or novice version of Western academic mathematical practices. Instead, mathematical activity is assumed to be situated as humans use social, cognitive, linguistic resources and cultural tools to make sense of problems.

In addition to ethno-mathematics, other research in mathematics education provides useful theoretical frameworks for integrating language into research on mathematics learning. As an example, two publications by Brenner provide theoretical distinctions that are relevant to both research and practice. Brenner (1994) provides useful distinctions among different kinds of communication in mathematics classrooms and describes three components of a "Communication Framework for Mathematics" as follows:

> "Communication About Mathematics entails the need for individuals to describe problem solving processes and their own thoughts about these processes . . .. Communication In Mathematics means using the language and symbols of mathematical conventions . . .Communication with mathematics refers to the uses of mathematics which empower students by enabling them to deal with meaningful problems." (Brenner, 1994, page 241)

The framework described by Brenner in another publication (1998) is useful for considering the relevance of research studies, organizing literature searches, and synthesizing across studies. Brenner (1998) provides a three-dimensional model for examining the cultural relevance of instruction and curriculum. This framework can be used to guide the design of research (as well as curriculum and instruction) by focusing research questions on aspects of classrooms that are relevant to three aspects of culturally relevant instruction:

1. <u>Cultural content</u>: Do the mathematical activities have a relation to the mathematical activities in the local culture?

2. <u>Cognitive resources</u>: Do instructional methods enable children to build from their existing cultural knowledge?

3. <u>Social organization</u>: Does the classroom community facilitate comfortable and productive participation of all students? Do the roles and responsibilities fit with learners' cultural models for communication?

The first and second questions have been addressed by research that examines the mathematical activities in local communities (for example González, Andrade, Civil, & Moll, 2001; Moll, Amanti, Neff, & González, 1992) and the cognitive resources students bring to the classroom for learning mathematics (for example algorithms across cultures Secada, 1983, Orey, 2003).

The third question is most relevant to issues of language. What constitutes comfortable and productive participation for different populations of mathematics learners who are bilingual, multilingual, and/or learning English? Sociolinguistics studies that have examined how young bilingual Latino learners use two languages (for example Zentella, 1997) provide a relevant knowledge base to address this question for young Latino learners. What cultural models for communication do students bring to the mathematics classroom? This question might be approached using research studies that have examined models for communication among particular student populations, for example native Hawaiian children (Au, 1980; Au and Jordan, 1981), Navaho students (Vogt, Jordan, & Tharp, 1987), and African-American children (Heath, 1983; Lee, 1993).

Empirical research on communication models for other student populations in the U.S. should also provide a relevant knowledge base for further research. However, research on communication models should be used only as examples of how communication practices can vary, rather than to make broad generalizations about the communication styles for any particular group of learners.

Many authors have warned researchers repeatedly about the danger in assuming that communication styles or home cultural practices are homogeneous in any community, dominant or non-dominant. For example, Gutierrez, Baquedano-Lopez, and Alvarez (2001) describe language practices as "hybrid" and based on more than one language, dialect, or practice. Rogoff and Gutiérrez (2003) also caution us against ascribing cultural practices to individuals and instead propose we consider the repertoires of practices that any one individual has had access to. We cannot assume that any cultural group has "cultural uniformity or a set of harmonious and homogeneous shared practices (González, 1995)." González (1995) decries perspectives that "have relegated notions of culture to observable surface markers of folklore, assuming that all members of a particular group share a normative, bounded, and integrated view of their culture" and suggests that "approaches to culture that take into account multiple perspectives can reorient educators to consider the everyday experiences of their student, " (González, 1995, page 237). Still, Brenner's three-part framework can be used as a guide to consider more broadly the complexity in what might constitute comfortable and productive participation for learners, as well as the varied practices that students have experienced across multiple settings. Researchers should keep in mind that learners from any community can and do participate productively in a variety of roles, responsibilities, communication styles, and mathematical activity that include hybrid practices.

3. Challenges in designing research on language and mathematics learning

Examining language and learning mathematics presents many interesting challenges. Barwell (2003) has described what he calls "linguistic discrimination," giving examples of how research studies sometimes assume homogeneity, position English as the norm and default language, or use written tests as the sole instruments to describe linguistic competence. In the next section I describe several other pervasive and enduring challenges: defining bilingualism, building on previous work, avoiding deficit models of bilingual learners, and avoiding superficial conclusions about language and cognition.

3.1 Defining bilingualism

The first challenge researchers face when designing research with bilingual/multilingual populations is how these labels are used in ambiguous ways and with multiple meanings. Research studies need to be clear in specifying how the labels 'bilingual' or 'multilingual' are used, both for classrooms and learners. There are many different labels for different types of bilingual classrooms and these labels do not always tell us what, exactly, happens in the classroom in terms of how teachers and students use each of the languages they may also use outside the classroom.

Studies should document and report not only students' proficiency in each language but also their histories, practices, and experiences with each language across a range of settings and tasks. Studies should describe proficiencies in each language wherever possible in both oral and written modes. There are serious challenges that such research will need to address, such as the complexity of defining a construct such as language "proficiency," the lack of instruments that are sensitive to oral and written modes, and the scarcity of instruments that address features of the mathematics register for specific mathematical topics.

Studies should not assess English or Spanish proficiency *in general* but rather, specifically for communicating in writing and orally about a particular mathematical topic. Students have different opportunities to talk and write about mathematics in each language, in informal or instructional settings, and about different mathematical topics. Assessments, then, should consider not only proficiency in each language, but also proficiency for using each language to talk or write about a particular mathematical topic. One example of how researchers might approach this challenge is Secada's (1991) study that used a complex view of language proficiency and examined the semantic structure of arithmetic word problems. Language proficiency was assessed using the Language Assessment Scales (De Avila & Duncan, 1981b; Duncan & De Avila, 1986 and 1987), oral story telling, and verbal counting up and down. The instruments assessed syntax, phonetics, lexicon, and pragmatics and included language tasks that were closely related to the specific mathematical thinking and topic examined in the study.

Studies should also document and report students' mathematical histories, practices, and experiences across a range of settings and tasks where mathematics is involved. It is important to examine mathematics learning in the context of its development while considering the tools and practices employed for mathematical reasoning. It is crucial that in documenting mathematical practices researchers consider the spectrum of mathematical activity as a continuum, rather than reifying the separation between practices in out of school settings, such as at home or on the playground and the practices at school. Lastly, research should consider mathematical activity from an emic perspective, as it is experienced by the participants, rather than imposing the view of a mathematical expert on the analysis of these practices. Neglecting these three aspects of mathematical practices typically leads to a deficit view of everyday mathematical practices.

Definitions of bilingualism range from native-like fluency in two languages, to alternating use of two languages (De Avila & Duncan, 1981a), to belonging to a bilingual community (Valdés-Fallis, 1978). Current scholars studying bilingualism see "native-like control of two or more languages" as an unrealistic definition that does not reflect evidence that the majority of bilinguals are rarely equally fluent in both languages (Grosjean, 1999). Grosjean proposes that, instead of comparing each language used by a bilingual against a monolingual norm, we focus instead on the ways in which *individuals* who use more than one language operate along a continuum of *modes*. Thus, depending on whether they are speaking to a monolingual or another bilingual, bilinguals make use of one language, the other language, or the two together as they move along a continuum from monolingual to bilingual *modes*.

One common misunderstanding of bilingualism is the assumption that bilinguals are equally fluent in their two languages. If they are not, then they have been described as not true, real, or balanced bilinguals and sometimes labeled as 'semilingual' or 'limited bilingual.' The concept of semilingualism has been discussed by several educational researchers and strongly criticized by many (for a review see Baetens Beardsmore, 1986 and MacSwan, 2000). In my own work, I use the definition of bilinguals provided by Valdés-Fallis (1978) as "the product of a specific linguistic community that uses one of its languages for certain functions and the other for other functions or situations" (p. 4). This definition describes bilingualism not only as individual but also as a social and cultural phenomenon involving participation in language practices and communities.

One solution to the use of vague and confusing labels is for researchers to know the students and the community well. In this way, they can make careful and informed decisions regarding what questions to ask in order to be able to describe the cultural and linguistic context clearly and in detail. There are many questions to ask about mathematics learners and these questions have complex answers: Do students themselves identify as bilingual or monolingual? Does the school identify students as monolingual or bilingual? If so, on what basis are students labeled bilingual or monolingual? Is a student a recent immigrant, first generation, second generation, or part of borderland communities? Do students come from rural, urban, migrant, farming settings? What are students' previous schooling experiences? How many years have they been in school? How much mathematics instruction have students experienced in each language? What are students' informal mathematical experiences with activities such as selling and buying, games, or work related mathematics?

3.2 Avoiding deficit models of learners and their communities

A crucial pitfall to avoid when examining language and mathematics learning for students who are bilingual, multilingual, or learning English is using deficit models of language minority learners and their communities. Many deficit models stem from assumptions about learners and their communities based on race, ethnicity, SES, and other characteristics assumed to be related in simple, and typically negative, ways to cognition and learning in general. For example, in the U.S. policy and research, rather than seeing bilingual Latino learners as having additional language skills have used deficit models to describe these students (Garcia & Gonzalez, 1995).

Deficit models are so pervasive and insidious, that we can sometimes fail to recognize them. For example, any time we use monolingual learners (or classrooms) as the norm, we are imposing a deficit model on bilingual learners. Bilinguals learning mathematics need to be described and understood on their own terms, and not only by comparison to monolinguals. Future research should move away from comparisons between monolingual and bilingual learners. Studies should focus less on comparisons to monolinguals and report not only differences between monolinguals and bilinguals, but also similarities. Studies focused on the differences between bilingual and monolinguals may miss the similarities, for example in how both types of students solve mathematics problems.

Early research focused on the disadvantages that English learners and bilinguals face, focusing on comparing response times between monolinguals and bilinguals (McLain and Huang, 1982; Marsh and Maki, 1976) or the obstacles the mathematics register in English presents for English learners (Spanos & Crandall, 1990; Spanos et al, 1988). Research has not yet seriously considered any possible advantages of bilingualism for mathematics learning. Future research should stop focusing on the disadvantages associated with learning English or being bilingual and explore any advantages that bilingualism might provide for learning mathematics. One example of an advantage for bilinguals is the reported role of attention in solving mathematical problems. After reviewing research on the cognitive consequences of bilingualism, Bialystok (2001) concluded that bilinguals develop an "enhanced ability to selectively attend to information and inhibit misleading cues (page 245)."[iv] This conclusion is based, in part, on the advantage reported in one study that included a proportional reasoning task (Bialystok and Majumder, 1998) and another using a sorting and classification task (Bialystok, 1999). Although these tasks seem closely related to mathematical problem solving, they have not been examined in detail in the context of bilinguals doing or learning mathematics.

Another common pitfall to avoid is blaming the parents, the community, or the culture for perceived deficits in the learners. This is a complex issue, best illustrated by an example. On the one hand, research has shown that parental education is related to student performance. For example, a study of 4th and 8th grade LEP (Limited English Proficiency) and non-LEP students in the U.S. found that students of parents with less than a high school education had lower average reading and math scores (Abedi, Leon, and Mirocha, 2003). Some LEPs with highly educated parents had higher scores than non-LEPs with parents with less than a high school education. However, it would be a simplistic application of a deficit model of the community to then conclude that the parents are the problem, and conclude that parents with less than a high school education perhaps do not care about education or do not help students with homework. In the case of Latino parents in the U.S., empirical research shows that this is simply not the case. Hispanic parents are more likely to attend PTA meetings and to help their children with homework than are white or African-American parents (Pew Hispanic Center, January 2004). A similar percentage of Latino and Anglo students reported parents regularly reviewed their homework (NCES 1995). Latino eight graders were more likely than their Anglo counterparts to report that parents had limited their TV viewing and that parents had visited their classes (NCES, 1995). Lastly, but perhaps most importantly, ethnographic studies repeatedly show that immigrant, Latino, and African American parents do, in fact, value education.

Focusing on the mathematical activity is another important way to avoid using deficiency models of bilingual learners. If research examining bilingual learners does not focus on the mathematical activity, then it may seem that bilingual learners do not engage in mathematical activity, thus further contributing to seeing them as deficient. It is crucial to uncover the mathematics bilingual learners are doing and that they are capable of doing. If analyses in mathematics education do not focus on the mathematical activity, then we contribute to a view that these learners are not *really* doing mathematics. Mathematics education research should keep the focus on the mathematical ideas and bring out the mathematics that bilingual and multilingual learners are engaged in.

In order to focus on the mathematical meanings learners construct, rather than the mistakes they make, researchers will need a framework for recognizing the mathematical knowledge, ideas, and learning that learners are constructing in, through, and with language. A functional theory of language such as functional systemic linguistics (for example O'Halloran, 1999 and Schleppegrell, 2007) and a communication framework for mathematics instruction (Brenner, 1994) can serve as frameworks for recognizing mathematical contributions by students.

A situated and socio-cultural perspective on bilingual mathematics learners (Moschkovich, 2002) shifts the focus from looking for deficits to identifying the mathematical discourse practices evident in student contributions (for an example, see Moschkovich, 1999). The socio-cultural perspective in Moschkovich (2004 and 2007b) also provides a theoretical framework for recognizing the mathematics in student contributions. This framework assumes that mathematical Discourse is complex, grounded in practices, and connected to mathematical concepts. I use the phrase "Discourse practices" to emphasize that Discourse is not individual, static, or referring only to language. Instead, I assume that meanings are multiple, situated, and connected to multiple communities. Discourses involve more than language, they also involve other symbolic expressions, objects, and communities. Discourse practices involve not only language but also perspectives and conceptual knowledge. Words, utterances, or texts have different meanings, functions, and goals depending on the practices in which they are embedded.[v] Discourses occur in the context of practices and practices are tied to communities. Discourse practices are both constituted by actions, meanings for utterances, focus of attention, and goals and that these actions, meanings, focus, and goals are embedded in practices.[vi] In previous publications I describe how mathematical practices involve goals, meanings for utterances, and focus of attention (Moschkovich, 2004) and examine how meanings for utterances reflect particular ways to focus attention (Moschkovich, 2007b).

3.3 Avoiding superficial conclusions about language and mathematical cognition

Another important pitfall to avoid is jumping to conclusions regarding language and cognition. Below I discuss two common conclusions about bilinguals and mathematical thinking made on the basis of observations of two common practices among bilingual mathematics learners, using two languages during mathematical conversations or when carrying out arithmetic computation.

One common practice among bilinguals is switching languages during a conversation, a phenomena called code switching. It is crucial to avoid reaching superficial conclusions regarding code switching and cognition, for example that bilinguals switch languages because they do not remember a word or do not know a concept. Regardless of what our personal experiences or folk explanations of code switching may tell us, empirical research in sociolinguistics has shown that code switching is a complex language practice and not evidence of deficiencies. In general, code switching is not primarily a reflection of language proficiency, discourse proficiency, or the ability to recall (Valdés-Fallis, 1978). Bilinguals use the two codes differently depending on the interlocutor, domain, topic, role and function. For example, young bilinguals (beyond age 5) "speak as they are spoken to" (Zentella, 1981). Last but not least, when using two codes the choice of one rather than the other involves an act of identity. Therefore it is not warranted to make simple conclusions between someone's code switching and their proficiency in a national language or in mathematics.

Another common practice among adult bilinguals is carrying out arithmetic computation in their first language and then translating the answer. A small set of psycholinguistic studies that have explored this phenomena are based on the hypothesis that since bilinguals tend to do mental arithmetic in their first language, therefore they take longer to compute. An example of this work comes from two studies conducted with adult US Spanish speakers (Marsh and Maki, 1976 and McLain and Huang, 1982). Marsh and Maki (1976) found that adult bilinguals performed arithmetic operations more rapidly in their preferred language than in their non-preferred language. The subjects (20 adult immigrants in the U.S. who preferred either English or Spanish) reported that that they performed arithmetic in their preferred language and then translated answers to the non-preferred language. The difference between performance in a preferred language and a non-preferred language was slight (200 milliseconds). Comparisons between monolinguals and bilinguals (who preferred English to Spanish) showed a slight but statistically significant difference of about .5 seconds for mean response time.

The evidence regarding bilinguals' speed during arithmetic computation is inconclusive and contradictory. All we can safely say at this time is that "retrieval times for arithmetic facts *may* be slower for bilinguals than monolinguals." (Bialystok, 2001, page 203.) This possible difference is incredibly small (on the order of .5 seconds) and has been documented clearly only with Spanish and Japanese speaking adults, not young children. It is not clear whether these reported small differences in response time between monolinguals and bilinguals exist for young learners. Overall, strong evidence suggests, "Language switching does not affect the quality and integrity of thinking at the conceptual level in second language production (Cumming 1989 and 1990, cited in Qi, 1998). In the words of one researcher summarizing these studies:

> "The results of these studies present a complex picture and appear in some instances to contradict each other. The most generous interpretation that is consistent with the data is that bilingualism has no effect on mathematical problem solving, providing that language proficiency is at least adequate for understanding the problem. Even solutions in the weaker language are unhampered under certain conditions" (Bialystok, 2001, page 203).

The result that seems most relevant to mathematics classrooms is that the reported slight difference in response time disappeared if bilinguals were not asked to switch languages during an experimental session. McLain & Huang (1982) showed that if bilinguals are required to use only one of their languages the "preferred language advantage" can be eliminated. This study supports classroom practices that allow bilingual students to choose the language they use for arithmetic computation in the classroom, rather than requiring them to change languages, because such changes may impact response time, even though this impact might be minimal.

3.4 Balancing building on and revising previous work

Another challenge we face when designing research on language and learning mathematics is finding ways to build on older work, and thus avoid reinventing wheels, while challenging and revising older work in light of new theory and data. We need to revisit early work in light of current research and also challenge outdated approaches when this is necessary. For example, early work on language and learning mathematics focused on how the mathematics register presented an obstacle for English learners (for examples see Spanos & Crandall, 1990; Spanos et al 1988) and did not examine any of the resources English learners bring. In contrast, more recent work considers not only the obstacles for learners of a second language, but also the resources learners bring to the task of learning mathematics in a second language (for an example see Moschkovich, 2002).

Cummins threshold hypothesis (1979) and the concept of "semilingualism' are examples of early work that should be revisited:

> "The obsession with monoglot norms of reference has led to the notion of semilingualism. Now the notion of semilingualism has led to considerable controversy and should be treated with great caution by anyone approaching bilingual studies" (Baetens Beardsmore, 1986).

Semilingualism was first introduced by Nils Erik Hansegard (a Swedish Philologist) in 1962 (in the absence of any theory of language) to conjecture that a period of "double semilingualism" occurs when an individual abandons her native language altogether in favor of a second language (MacSwan, 2000). In the U.S. Cummins (1976) used the term in describing the *Threshold Hypothesis*, the hypothesis that the level of linguistic competence attained by a bilingual child in first and second language may affect his or her cognitive growth in other domains. Cummins defined "semilingualism" as low level in both languages (Cummins, 1979) to describe students who do not develop "native-like competence in either of their two languages" (Cummins, 1976, p. 20). This definition involves the conjecture that some children have limited or nonnative ability in the language or languages they speak (MacSwan, 2000).

Currently, most scholars have discarded the concept, even Skutnabb-Kangas and Cummins, two of the early proponents of this notion:

> "I do not consider semilingualism to be a linguistic or scientific concept at all. In my view, it is apolitical concept.' (Skutnabb-Kangas, 1984, p. 248)
>
> "There appears to be little justification for continued use of the term "semilingualism" in that it has not theoretical value and confuses rather than clarifies the issue" (Cummins, 1994, p. 3813).

The grounds for discarding the concept range from its nebulous nature to a lack of empirical support and theoretical foundation for the notion:

> Semilingualism does not exist, or put in a way which is non-refutable, has never been empirically demonstrated. " (Paulston, 1982, p. 54)

Critics of the concept of "semilingualism" argue that it confounds language proficiency (or linguistic competency) and use of academic register, formal schooling, SES, or "language loss" (the shift in choice of language occurring across generations). The concept also confuses degrees of ability, levels of linguistic competence, or levels of language development with differences in experience with language varieties (dialects, registers, and discourses) or with school literacy (reading, writing, and other aspects of language use valued in school).

Perhaps the strongest argument against semilingualism is the empirical evidence that it is not possible to have limited or nonnative ability in the language of one's own home community. Linguists agree that "all normal children acquire the language of their speech community with some minor but ordinary degree of variation" and that "a native language is acquired[vii] effortlessly and without instruction by all normal children" (MacSwan, 2000, p. 25)[viii].

4. Methodological issues in designing research on language and learning mathematics

While the term "methodology" is sometimes used to refer only to "methods," I use the term "methodology" to refer to theory and methods together because theory and methods are mutually constructive (Moschkovich, and Brenner, 2000). I assume that methodology includes the underlying theoretical assumptions about cognition and learning: what cognition and learning are; when and where cognition and learning occur; and how to document, describe, and explain these phenomena. All the issues raised in this section pertain not only to the methods one uses to examine language and mathematics learning, but, more fundamentally, to the theoretical way that one frames and conceives of both mathematical activity and language.

There are methodological issues that are specific to designing and conducting research on language and learning mathematics. Most importantly, research needs to address and focus on the needs of specific student populations, rather than using categories such as English learners, bilinguals, or Latinos as if these groups were homogeneous. 'English Learners' is a particularly vague label that does not capture the linguistic complexity of student experiences when learning mathematics. This category can include multiple student populations and collects learners into one category even thought they have different experiences and needs, for example Latinos who are monolingual, Latinos who are bilingual, Asian students, immigrants, young children, adolescents, etc.

4.1 Data collection

When designing research on language and learning mathematics it is important to consider what data to collect, which tools to use and how, and how to assess language proficiency. In general, the design of data collection should consider and build on the instruments used in previous research literature that is relevant, such as assessments of language proficiency in a particular topic in Spanish (for example whole number operations in Secada, 1991) or assessments of reading proficiency in English that use traditional word problems (for example Clarkson and Galbraith, 1992).

The analysis of classroom activity should be couched and framed by many types of data: the teacher's goals, textbook use, district policies, preceding lessons, information about the students, etc. This is especially important for cross-cultural work:

> "Contextualizing recorded behavior is important in understanding the meaning behind that behavior, and it is especially important when coding across cultures (Erickson, 1986). Similar behaviors may have different meanings and comparisons can be problematic." (Ulewicz & Beatty, 2001, page 13)

Research studies need to provide detailed ethnographic details about students' language backgrounds and experiences with mathematics in and out of school. Important information to collect when working with bilingual populations includes whether students are immigrants, first generation, or second generation. Information about previous schooling experiences is also important, such as whether students have participated in mathematics classes in their first language or not. Information on previous mathematical experiences out side of school, such as selling, buying, games, and so on, is also relevant. Research should document and report not only information about students' proficiency in each language but also their experiences with each language and with mathematics at home and at school. Assessing language proficiency is complex and involves oral, written, listening, and academic registers in content areas. Studies should distinguish between proficiencies in oral and written modes and, wherever possible, describe proficiencies in each language in more than one mode (oral and written).

It seems especially important to consider *how* we might use video data to examine language, mathematical activity, and mathematics learning with bilingual student populations, particularly for evaluative analyses of student activity. It is a common experience when analyzing video data to focus on what a student is doing wrong rather than on how what a student is doing well. Because video slows action down, participants on videotape may seem both less and more competent than in real time. As we watch video we have more time to notice how participants mis-peak or make mistakes than we would have if we were observing in real time, thus making them appear *less* competent. This tendency to focus on the negative when looking at videotapes is a special danger when we analyze bilingual, multilingual, or English learners' mathematical activity. When looking at video data of bilingual or multilingual learners, it is especially important to not equate a participant's linguistic competence with competence in mathematical reasoning. On the other hand, as we watch video we also have more time to notice and really think about what participants said and did, potentially making them look *more* competent than in real time. Thus, video data also opens up the possibility to document student competence in mathematical reasoning.

4.2 Transcription and translation

There are several challenges involved in working with more than one national language. Two important issues for research that integrates issues of language and mathematics learning are transcriptions and translations.

Transcription and transcript quality are theory laden (Ochs, 1979, Poland 2002). Researchers make many decisions about transcripts that are based on their theoretical framework and on the particular research questions for a study. For example, decisions regarding what to include in transcripts and which transcript conventions to use are informed by theory. Whether a transcript will include gestures, emotions, inscriptions, body posture, and description of the scene (Hall, 2000; Poland, 2002; McDermott and Gospodinoff, 1978), will depend on whether these aspects of activity are relevant or not to the particular research questions. Similarly, selecting transcript conventions and deciding whether overlapping utterances, intonation, and pauses are included or not in a transcript depends on whether these aspects are relevant to the research questions and analysis that the transcript and video will be used for. And whether and how aspects of activity are relevant (or not) to the research questions depends on the theoretical framework.

Two aspects of bilingual conversations and features of talk that may be relevant to documenting mathematics learning are intonation and the use of gestures. Perceiving a student as uncertain or hesitant because of intonation patterns may have an impact on how researchers (and teachers) perceive student contributions in mathematics classrooms. For example, intonation patterns vary across languages and among dialects:

> "Perhaps the most prominent feature distinguishing Chicano English from other varieties of American English is its use of certain intonation patterns. These intonation patterns often strike other English speakers as uncertain or hesitant" (Finegan & Besnier, 1989, p. 407).

Bilingual students' use of gestures to convey mathematical meaning has been documented (for example, Moschkovich 1999, 2002). Further exploration of the use of gestures during mathematical discussions would provide more detailed descriptions of the role of gestures and intonation in how bilingual/multilingual mathematics learners communicate.

Translation presents a challenge all its own. Translation is not simply a copy of the original utterance since translating involves interpretation. It is impossible to translate without putting some piece of ourselves in the new utterance– translators are not simply empty vessels. When participants use two languages, it is important for researchers to choose clear ways to display transcripts and decide how the transcript will show translations. It is imperative that both the actual utterances of the participants in one (or more) language(s) as well as the translations be included in presentations and publications reporting on the research. Subtitles can be a useful way to display both an utterance and its translation, allowing other researchers to inspect both simultaneously.

Needless to say, translations should be done as carefully as transcriptions. Transcripts in another language should be as accurate as English transcripts, using not only the skills of native speakers but also professional translators. Knowing the particular community of students and checking translations with native speakers of the particular regional dialect are crucial aspects of translation work. Translators need to pay attention to the way that language is used in each particular community. Ideally, translators need to have knowledge of regional and national variation and know the community and the students. For example, transcribers and translators need to be knowledgeable about the particular variety of Spanish (for example Puerto Rican or Mexican-American) so that words, phrases and expressions particular to each community are accurately translated. Researchers who do not speak the second language fluently themselves will need to be aware of whether a transcriber or translator is fluent in particular dialects of that language and has experience using the mathematics register. An important question to consider for translations of mathematical activity in more than one language is how domain knowledge impacts translation. Ideally, translators need to have mathematical knowledge and know mathematical terms and expressions in both languages. Involving a professional translator and using a second translator to check are useful strategies for managing the many challenges of translating children's mathematical discussions in two languages.

5. Closing

In closing, I describe a few general areas categories of research questions that seem fruitful for future research to consider. In order to be able to design instruction that builds on student resources, research needs to examine in more detail the resources that bilingual or English learning students use for mathematical reasoning. Many more studies are needed that describe how students who speak more than one language use multiple resources such as two languages, gestures, objects, and mathematical representations or inscriptions to communicate mathematically. Studies will need to distinguish among multiple modalities (written and oral) as well as between receptive and productive skills. Other important distinctions are between listening and oral comprehension, comprehending and producing oral contributions, and comprehending and producing written text.

It is also important for research to move away from construing everyday and school mathematical practices as a dichotomous distinction. During mathematical discussions students use multiple resources from their experiences across multiple settings, both in and out of school. Everyday practices should not be seen only as obstacles to participation in academic mathematical Discourse. The origin of some mathematical Discourse practices may be everyday practices and some aspects of everyday experiences can provide resources in the mathematics classroom. Everyday experiences with natural phenomena can be resources for communicating mathematically. For example, climbing hills is an experience that can be a resource for describing the steepness of lines (Moschkovich, 1996). Other everyday experiences with natural phenomena also may provide resources for communicating mathematically.

In addition to experiences with natural phenomena, O'Connor (1999) proposes that students' mathematical arguments can be at least partly based on what she calls argument proto-forms:

> Experiential precursors (arguments outside of school, the provision of justification to parents and siblings, the struggle to name roles or objects in play) may provide the discourse "protoforms" that students could potentially build upon in the mathematical domain. (p. 27, O'Connor, 1999)

These precursors are related to academic mathematical Discourse practices such as arguing, making and evaluating a claim, providing justification, or co-constructing a definition. Research should consider what aspects of everyday discourse could serve as resources for mathematical arguments.

Academic English (Scarcella, 2003) for mathematical communication is another important topic for research studies to address. Studies need to examine in more detail what, exactly, constitutes Academic English competency for mathematics, in both written and oral modes. If it is the case that Academic English is different for different mathematical domains or genres of mathematical texts, then these differences need to be examined. Studies also need to explore how immigrant students transition from learning mathematics in Spanish to learning mathematics in English: What are students' experiences learning mathematics in their first, second, and both languages? How do different proficiencies in a first language (oral, reading, written, academic English) and previous mathematics instruction in a first language impact students' learning mathematics in English?

Learning to read and use vocabulary in mathematics are two topics that also need attention from researchers. Studies are needed that examine how English learners learn to read different mathematical texts (textbooks, word problems, etc.). In designing this research it will be important to differentiate between reading textbooks and reading word problems, two different genres in mathematical written discourse. When working with children who are learning to read in English, it will also be important to distinguish between children who are competent readers in a first language and those children who are not. Research also needs to consider what are successful ways for English learners to learn vocabulary in mathematics. This work will need to start by establishing what vocabulary assessment instruments are relevant to English learners or bilingual students learning mathematics.

The question is not whether students should learn vocabulary but rather how instruction can best support students learning both vocabulary and mathematics. Vocabulary drill and practice is not the most effective instructional practice for learning either vocabulary or mathematics. Instead, vocabulary and second language acquisition experts describe vocabulary acquisition in a first or second language as occurring most successfully in instructional contexts that are language rich, actively involve students in using language, require both receptive and expressive understanding, and require students to use words in multiple ways over extended periods of time (Blachowicz & Fisher, 2000; Pressley, 2000). We already know that to develop written and oral communication skills students need to participate in negotiating meaning (Savignon, 1991) and in tasks that require output from students (Swain, 2001). Researchers in vocabulary acquisition agree that the best way for students to develop mathematical vocabulary is to provide opportunities for them to actively use mathematical language to communicate about and negotiate meaning for mathematical situations. It is especially important that instruction for this population not emphasize low-level language skills over opportunities to actively and repeatedly communicate about mathematical ideas. One of the goals of mathematics instruction for bilingual students should be to support all students, regardless of their proficiency in English, in participating in discussions that focus on important mathematical ideas, rather than on pronunciation, vocabulary, or low-level linguistic skills.

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[i] My intention here is not to provide an exhaustive literature review, but rather to mention a few selected and relevant studies. More extensive, exhaustive, and detailed reviews of the literature can be found in Adler, 2001; Moschkovich, 2002, 2007, in review; Clarkson, 1991; Ellerton & Clements, 1991.

[ii] Because this essay focuses on children learning mathematics, I do not include here research documenting teachers code switching in classrooms. For examples, see Khisty (1995), Setati (1998), Setati & Adler (2001), Valdés-Fallis (1976), and Zentella (1981).

[iii] There is also a small percentage of Latino children and adults in the U.S. who also speak an indigenous language as their first language, Spanish as a second language, and English as a third language.

[2] Bialystok mentions that the cognitive advantages of bilingualism seem to depend on some level of proficiency in both languages. (Bialystok, 2001, page 205)

[v] I use the terms *practice* and *practices* in the sense used by Scribner (1984) for a practice account of literacy to "highlight the culturally organized nature of significant literacy activities and their conceptual kinship to other culturally organized activities involving different technologies and symbol systems" (p.13).

[vi] For a description of how discourse practices involve actions and goals and an analysis

of the role of goals in the appropriation of mathematical practices, see Moschkovich (2004).

[vii] The notion of acquiring a language does not imply that once one acquires the language of their community this is a static end in language learning. The discussion throughout this essay should make it clear that language acquisition and proficiency are nuanced, situated, and dynamic processes.

[viii] While setting aside the notion of semilingualism, some researchers agree that variation exists in students' proficiency in educationally relevant aspects of language, in the formal language skills in one or more languages, and that bilingual learners need to develop is currently called "academic English." The topic of academic English, however, is beyond the scope of this essay (for a discussion of academic English see Cummins, 2000).